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Iterated Choquet expectations: A possibility result

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HIGHLIGHTS

- The law of iterated expectations is proven for Choquet preferences.
- The analysis is constrained to fixed partitions.
- The additive separability condition of an unconditional capacity and Bayesian updating are necessary and sufficient.
- Behavioral axioms justifying the law of iterated Choquet integrals are provided.

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ABSTRACT

Contrary to Yoo's (1991) [Yoo, K.-R., 1991. The iterative law of expectation and non-additive probability measure. Economics Letters 37, 145–149] result, it is shown that the law of iterated expectations can be maintained in the class of Choquet expected utility preferences, even though beliefs are non-additive. Choquet integrals satisfy the law of iterated expectations on a fixed partition if and only if the unconditional capacity is additively separable among the events forming that partition, and the conditional capacity is derived by applying Bayes' rule.

1. Introduction

A broad range of economic decision models such as auction, asset pricing, or investments involves a dynamic acquisition of information, whether it is public or private. For a given information structure, the law of iterated expectations ensures that the time at which the information is received is immaterial for future decisions. Under the law of iterated expectations, a decision maker's expectation of a random variable given information today is equal to the expected value of his/her conditional expectations of the random variable taken with respect to information tomorrow. This property is extremely useful in solving dynamic optimization problems under uncertainty; it enables us to apply procedures, such as backward induction, and to evaluate recursively dynamic decisions.

Under the Bayesian paradigm, the law of iterated expectations is guaranteed by the properties of additive probability measures and Bayesian updating (see Billingsley, 1995). However, as exemplified by Ellsberg (1961), subjects facing decision problems under ambiguity, i.e. situations where probabilities for uncertain events are unknown, may be incapable of forming beliefs satisfying the standard properties of probability measures. In particular, their beliefs may be non-additive. One prominent theory that accommodates behavior with non-additive beliefs is the Choquet expected utility theory of Schmeidler (1989). In this theory, expectations are computed as Choquet integrals (due to Choquet, 1954) with respect to non-additive measures called capacities. Schmeidler's theory has been successfully applied to a wide range of economic theories such as game theory (e.g. Dow and Werlang, 1994; Eichberger and Kelsey, 2000), contract theory (Mukerji, 1998), auction theory (Salo and Weber, 1995), financial markets (e.g. Dow and Werlang, 1992; Mukerji and Tallon, 2001), and speculative trade (Dominiak et al., 2012), providing predictions that differ from those derived in Bayesian setups. In order to make the Choquet expected utility theory more tractable with applications to dynamic decision problems, it is important to clarify whether the law of iterated expectations can be preserved within the theory, and, if it can be maintained, what the conditions characterizing the law are.

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Yoo (1991) asserts that the law of iterated expectations cannot be sustained in the class of Choquet preferences unless beliefs are additive. Yoo proves that Choquet integrals satisfy the law of iterated expectations if and only if capacities coincide with additive probability measures (i.e., preferences are of the subjective expected utility type of Savage, 1954). This note demonstrates that one can allow for non-additive beliefs and still maintain the law of iterated Choquet expectations by constraining the analysis to fixed partitions. We characterize the properties of the unconditional capacities and of an updating rule that are necessary and sufficient for Choquet preferences to satisfy the law of iterated expectations on fixed partitions. We also refer to static and dynamic axioms that behaviorally justify the law of iterated Choquet expectations on fixed partitions.

2. Formal setup

There is a finite set $\Omega = \{\omega_1, \dots, \omega_N\}$ of states with $N \geq 4$. An event E is a subset of Ω. For any $E \subseteq \Omega$, the complementary event is denoted by E^c. A capacity $\nu : 2^\Omega \to \mathbb{R}$ is a normalized and monotone set function. Let X be a set of outcomes. Denote by $\mathcal{F}$ the set of all acts (i.e., functions $f : \Omega \to X$). A constant act $f = x$ assigns an outcome $x \in X$ to all states. A binary act $f = xEy$ assigns a constant act x to all states in E, and y to states in E^c. Denote by $\succcurlyeq$ the preference relation on $\mathcal{F}$. An event E is called null (respectively, universal) with respect to $\succcurlyeq$ if $y \sim xEy$ (respectively, $x \sim xEy$) for all $x, y \in X$ such that $x \succ y$. An event E is called essential if it is neither null nor universal. The preference relation $\succcurlyeq$ admits a Choquet expected utility representation. There exist a utility function $u : X \to \mathbb{R}$ and a capacity ν on 2^Ω such that, for all acts $f, g \in \mathcal{F}$,

$$f \succcurlyeq g \iff \int_\Omega u(f)\, d\nu \geq \int_\Omega u(g)\, d\nu. \tag{1}$$

For a given u, the Choquet integral of $f \in \mathcal{F}$ with respect to ν is defined to be

$$\int_\Omega u(f)\, d\nu = \sum_{n=1}^{N} u(x_n) \left[\nu(\cup_{i=1}^{n} E_i) - \nu(\cup_{i=1}^{n} E_{i-1})\right], \tag{2}$$

where $u(f(\omega_1)) \geq \cdots \geq u(f(\omega_1))$ and $\nu(E_0) = \nu(\emptyset)$. We require $\succcurlyeq$ and X to fulfill solvability: for each $f \in \mathcal{F}$, there exists an $x \in X$ such that $f \sim x$. The solvability assumption is maintained in all axiomatizations of Choquet preferences in the Savage-style framework with a finite state space; e.g., see Wakker (1989), Nakamura (1990), and Chew and Karni (1994).

Let Π be a partition of Ω, and let A be an element of Π. It is assumed that each $A \in \Pi$ is essential. Denote the family of conditional preferences over $\mathcal{F}$ by $\{\succcurlyeq_A\}_{A\in\Pi}$, each derived upon $A \in \Pi$. Given an event $A \in \Pi$, the conditional preference relation $\succcurlyeq_A$ is representable by (1) with respect to the unconditional utility function u and an updated capacity ν_A. The conditional Choquet preferences are supposed to respect consequentialism, a property of preferences introduced by Hammond (1988, 1989). Consequentialism requires that, for any two acts $f, g \in \mathcal{F}$, it is true that $f \sim_A g$ whenever $f(\omega) = g(\omega)$ for all $\omega \in A$. Consequentialism intuitively states that conditional preferences are only affected by states in conditioning events, whereas counter-factual states remain immaterial for future choices.[1] There are many reasonable revision rules which respect the property of consequentialism, including the class of Gilboa and Schmeidler's (1993) f-Bayesian updating rules. The focus in this note is placed on one prominent example of f-Bayesian updating rules, the standard Bayes rule. For any $A \in \Pi$ and $E \subseteq \Omega$, the Bayesian update of ν given A is defined to be $\nu_A(E) = \frac{\nu(A\cap E)}{\nu(A)}$. We denote the family of conditional capacities by $\{\nu_A\}_{A\in\Pi}$, each derived upon $A \in \Pi$.

3. Result

Below, we fix a partition $\Pi = \{A_1, \dots, A_H\}$ of Ω and examine the properties of the events of that partition and of an updating rule that are necessary and sufficient to maintain the law of iterated expectations for Choquet expected utility preferences on Π. As Example 1 illustrates, the additivity of a capacity, constrained to the events forming partition Π, and Bayesian updating are not sufficient for the law of iterated Choquet integrals to hold true on Π.

Example 1. Let $A = \{\omega_1, \omega_2\}, A^c = \{\omega_3, \omega_4\}$ be the partition of $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. The capacity ν on 2^Ω is defined as follows: $\nu(A) = \nu(A^c) = 0.5$; and for $E \neq A, A^c$; $\nu(E) = 0.1$ if $|E| = 1$; $\nu(E) = 0.3$ if $|E| = 2$; $\nu(E) = 0.6$ if $|E| = 3$. Define $E = \{\omega_1, \omega_3\}$. Consider the binary act f paying off 1 on E, and 0 on E^c. Suppose that $u(x) = x$. The unconditional Choquet integral of f with respect to ν and u amounts to 0.3. However, the conditional capacities given A and A^c obtained by Bayesian updating yield the same Choquet integrals, which equal 0.2. Thus, the law of iteration is violated.

However, when the additivity of a capacity is strengthened to the additive separability condition introduced by Nehring (1999), then Choquet integrals do respect the law of iterated expectations under Bayesian updating. A capacity ν is said to satisfy the additive separability condition on partition Π if, for any $E \subseteq \Omega$,

$$\nu(E) = \sum_{h=1}^{H} \nu(E \cap A_h). \tag{3}$$

For Nehring (1999), a decision maker with subjective beliefs satisfying condition (3) perceives the events forming partition Π as being unambiguous, while other events may be regarded as being ambiguous. According to Nehring, an event is revealed to be unambiguous if the measure assigned to the event in Choquet integral (2) is independent of the rank of the event. In other words, the measure attached to the unambiguous event is always the same, regardless of which act is being evaluated.[2] The additive separability condition which has been imposed on partition Π together with Bayesian updating is not only sufficient but also necessary for the law of iterated Choquet expected utilities to remain valid on Π. It is formally stated in Theorem 1.

Theorem 1. *Fix a partition $\Pi = \{A_1, \dots, A_H\}$ of Ω. Let ν be an unconditional capacity on 2^Ω, and let $\{\nu_A\}_{A\in\Pi}$ be a family of conditional capacities. Then, the following two statements are equivalent.*

(i) *For each $f \in \mathcal{F}$,*

$$\int_\Omega u(f)\, d\nu = \int_\Omega \left(\int_A u(f)\, d\nu_A\right) d\nu. \tag{4}$$

[1] Hanany and Klibanoff (2007, 2009) provide a very comprehensive explanation of consequentialism.

[2] The concept of "rank-independent" measures as a definition of (exogenously) unambiguous partitions was suggested by Sarin and Wakker (1992) in their axiomatization of the Choquet expected utility preferences in a purely subjective framework.