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The role of information in contests[☆]

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HIGHLIGHTS

- The intensity of competition in a contest depends upon information regarding rivals.
- Incomplete information creates more competition, if the prize is of high value.
- Complete information creates more competition, if the prize is of low value.

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ABSTRACT

Consider a contest for a prize in which each player knows his/her own ability, but may or may not know those of his/her rivals (the complete or incomplete information regimes). Our main result is that, if the value of the prize is high, more effort and output are engendered under incomplete information, whereas, if the value is low, that distinction goes to complete information.

1. Introduction

Consider a contest in which a prize is awarded to the highest output. Each player's output depends upon his/her innate ability, which he/she is presumed to always know, as well as the effort he/she undertakes. We consider two information regimes: "complete information", where each player also knows the abilities of his/her rivals; or "incomplete information", where each knows only the probability distribution on their abilities, not the actual realizations. We show that, if the value[1] of the prize is high, then more competition, and hence also more effort and output, is engendered under incomplete information. In contrast, if the value is low, then that distinction goes to complete information.

The intuition behind the result is extremely simple, and is brought out with minimal fuss in a binary world[2] of two players, who could have either low or high ability, and who could "shirk" or "work". If both have similar abilities, then the contest will be evenly poised, and each player will find it worth his/her while

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1 We assume similarity in players' valuations, i.e., everyone unanimously accords a high (or low) value to the prize. ("Mixed" valuations are not considered in this paper, though our techniques can be applied to that case as well.)

2 But we shall emphasize only those results, and methods of proof, that go beyond the binary world. (See Dubey, 2012 for a heuristic discussion of various extensions of our model.)

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to work since he/she gets half a shot at the prize. But if their abilities are *widely* disparate, then the weak (strong) will have a low (high) probability of winning the prize, regardless of the effort levels the two have chosen, with the upshot that neither will have incentive to work. We conclude that, under complete information, both will work if they have similar abilities, and they will shirk if they are disparate. Now let us turn to incomplete information. Here, a player must choose his/her effort level conditional only on his/her own ability, since he/she does not know those of his/her rivals. He/she ascribes positive probability to the event that the rival is similar to him/her, and – as was said – in this event it pays for him/her to work. Thus, if the value of the prize is sufficiently high, his/her *expected* gain from effort overcomes the cost of that effort, even though the effort goes to waste when the rival is disparate. Thus each player works unconditionally (regardless of his/her ability). By the same argument, the situation is reversed for a low prize: the expected gain is less than the cost incurred from work, so both shirk unconditionally.

Of course – for low prizes – matters can become a bit more delicate when the disparity in abilities is not too wide, because in this case the players may have incentive to work, even when the prize is of low value and there is incomplete information. However, as the analysis below will show, our result is not marred when we average on a domain of ability pairs that is "sufficiently" diverse.

Related literature. There is a vast literature on contests for prizes, under conditions of either complete or incomplete information (by way of a list that is indicative, but by no means exhaustive, see Lazaer and Rosen, 1981, Green and Stokey, 1983, Rosen, 1986, Glazer and Hassin, 1988, Barut and Kovenock, 1998, Krishna and Morgan, 1998, Moldovanu and Sela, 2001, and the references therein). But, to the best of our knowledge, there has been no comparison of the two information regimes from the standpoint of generating more competition and output.

There is also work – related in spirit – on the value of public information in a Cournot duopoly; see in particular (Einy et al., 2002) and its references. Apart from the special structure of the Cournot model, and several technical differences that are discussed in detail in Dubey (2012), the biggest difference lies in the point of view: they consider participants' payoffs, whereas we focus on total output, though of course the two goals may sometimes be in tandem.

2. The binary model

There are two players who, for simplicity, are assumed to be risk neutral and ex ante symmetric. Each *player* $i \in \{1, 2\}$ can have one of two *abilities*[3] $a \in \{\alpha, \beta\}$ and can choose one of two *effort levels* $e \in \{0, 1\} = \{\text{shirk, work}\}$. If i has ability a, then the *output* produced by i via effort 0, 1 is given by a, $k(a)a$, respectively, for some $k(a) > 1$. (This serves to *define* a and $k(a)$.)

The abilities α, β are picked independently for the two players with probabilities π, $1-\pi$. For both players, the cost of effort 0 is 0 and that of effort 1 is $c > 0$. Both place the same value $v > 0$ on a *prize*, which is awarded to the player with the higher output, or randomized equally in the case of a tie.

Scaling c and v by the same positive factor is tantamount to a change of units in measuring players' payoffs, and leaves the contest unchanged, so, without loss of generality, we fix c and vary v. (We *could* vary the function k too, but do not do so, in order to keep the exposition simple.)

This well defines the *games* $\Gamma_C(\alpha, \beta, \pi, v)$, $\Gamma_I(\alpha, \beta, \pi, v)$ of *complete, incomplete information*, respectively.

[3] Both α and β are positive scalars.

The game $\Gamma_C(\alpha, \beta, \pi, v)$ is made up of four constituent 2×2 bimatrix *subgames* corresponding to the pairs (α, α), (β, β), (α, β), (β, α), which occur with probabilities π^2, $(1-\pi)^2$, $\pi(1-\pi)$, $(1-\pi)\pi$, with players similar in the first two subgames and disparate in the other two. Here, a player's pure actions are to shirk or work, and so a pure strategy in $\Gamma_C(\alpha, \beta, \pi, v)$ is a map from the four ability pairs to the two actions (i.e., a choice of action in each subgame).

As for $\Gamma_I(\alpha, \beta, \pi, v)$, it is a 4×4 bimatrix game, in which each player has the pure strategies $\{0, 0\}$, $\{1, 0\}$, $\{0, 1\}$, $\{1, 1\}$, where $\{e, f\}$ means that he/she chooses effort e when his/her ability is α, and f when it is β.

We shall vary all the parameters α, β, π, v of our model, since our interest lies not so much in any specific game as in the global behavior over a diverse domain of games. To this end, let $A \subset \mathbb{R}_{++}$ and $P \subset (0, 1)$ be arbitrary closed intervals of positive length. And, for any v, let $(\alpha, \beta, \pi) \in A \times A \times P$ be distributed according to some measure λ, which has a strictly positive density throughout $A \times A \times P$.

Our assumptions on the binary model are as follows.

Axiom 1 (*Minimum Valuation*)**.** $v > 2c$.

This says that, were the prize to be split equally, both players would work (since $v/2 > c$). The axiom thus enables us to focus on the failure of work occasioned by strategic competition, rather than inadequacy of the prize.

Axiom 2 (*Monotonicity of Output*)**.** $k(a)a$ is strictly monotonic in $a \in A$.

In other words, the output of work goes up as ability increases.

Axiom 3 (*Sufficient Disparity*)**.** There exists $(\alpha, \beta) \in A \times A$ such that $k(\alpha)\alpha < \beta$.

The mild requirement here is that A be sufficiently diverse to admit a "widely disparate" pair $(\alpha, \beta) \in A \times A$, where α is so weak relative to β that he/she lags behind β even if he/she works and β shirks.

3. Main result

For a clean statement of our result, we trim $A \times A$ a bit, by removing a negligible subset. Let $D = \{(\alpha, \beta) \in A \times A : \alpha = \beta\}$, and $E_1 = \{(\alpha, \beta) \in A \times A : k(\alpha)\alpha = \beta\}$, and $E_2 = \{(\alpha, \beta) \in A \times A : k(\beta)\beta = \alpha\}$. Thus D is the "northeasterly" diagonal of the square $A \times A$, and E_1 is a monotonic curve above D (the intersection of $A \times A$ with the graph of the function $k(a)a$ defined on domain A), and E_2 is the reflection of E_1 around D. Removing these three curves from the square, we put $R = (A \times A) \setminus (D \cup E_1 \cup E_2)$.

Let us define our *space of games* to be $\Sigma \equiv R \times P$, which differs from $A \times A \times P$ only by a λ-null set.

All the games in the space Σ have unique *Nash equilibria* (NE). (Indeed, as we shall see, quite often the NE are in *strictly dominant strategies* (SD).) Taking this fact provisionally on faith, denote the average output in $\Gamma_C(\alpha, \beta, \pi, v)$ and $\Gamma_I(\alpha, \beta, \pi, v)$ at the NE by $\tau_C(\alpha, \beta, \pi, v)$ and $\tau_I(\alpha, \beta, \pi, v)$. Then, the overall output on Σ engendered by a prize of value v under the complete and incomplete information regimes is given by

$$\tau_C(v) = \int_{\Sigma} \tau_C(\alpha, \beta, \pi, v) d\lambda(\alpha, \beta, \pi)$$

and

$$\tau_I(v) = \int_{\Sigma} \tau_I(\alpha, \beta, \pi, v) d\lambda(\alpha, \beta, \pi).$$

We are ready to state our main result.