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A note on Futia (1981)'s non-existence pathology of rational expectations equilibria☆

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HIGHLIGHTS

- We resolve the non-existence pathologies of Futia (1981).
- Knowledge of the model structure overturns the non-existence.
- The non-existence result is in fact a Full Communication equilibrium.

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ABSTRACT

We resolve the non-existence pathologies of dynamic rational expectations equilibria attributed to signal extraction from endogenous variables first discovered by Futia (1981). Non-existence is overturned once it is recognized that rational agents take into account the structure of the model when generating equilibrium outcomes. We show that where Futia (1981) thought an equilibrium did not exist, a Rational Expectations equilibrium identical to the Full Communication equilibrium does exist, thereby resolving the long-standing non-existence pathology.

1. Introduction

In a seminal paper in the January 1981 issue of *Econometrica*, Carl Futia characterized rational expectations equilibria in the presence of incomplete information and where equilibrium variables provided endogenous information to economic agents. The problem is non-trivial since the rational expectations equilibrium requires finding a fixed point in both optimal strategies and information sets. In standard representative agent economies these fixed point conditions are well understood and are easy to compute. When agents have incomplete information and extract information from endogenous variables, the fixed point conditions are more challenging to construct.[1]

The final message of Futia (1981) was mixed. On the one hand, Futia showed in the context of a simple model that a rational expectations equilibrium where all the agents are symmetrically informed is always identical to the unique Full Communication (FC) equilibrium, (i.e. the equilibrium that would emerge if all the available information in the economy was pooled and given exogenously to the agents). On the other hand, he also showed that this is true only if the stochastic process characterizing the FC equilibrium obeys a certain invertibility property. When the invertibility property is not satisfied, he argued that there cannot be a rational expectations equilibrium and hence the non-existence pathology.

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[1] Townsend (1983) and Sargent (1991) contain informative discussions of the issues.

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The non-existence pathology undoubtedly casts a feeling of uneasiness in dealing with dynamic rational expectations models with incomplete information. Arguably, the response of the subsequent literature to the troubling message of Futia (1981) has been to structure the information set with as much exogenous information as necessary to side-step the existence issue (Atkeson, 2000). In doing so, information is taken out of the hands of the equilibrium interactions and put into the hands of mechanical signal extraction algorithms. This modeling choice, while ensuring existence, has the unintended consequence of mooting potentially interesting insights stemming from the subtle ways in which the information is shaped by the equilibrium forces.[2]

In this paper, we show that the non-existence pathology in Futia (1981) is a consequence of an inconsistency between the definition of rational expectations equilibria employed in Futia's analysis and the argument he used in proving that a rational expectations equilibrium must always be identical to the FC equilibrium. We show that, the definition of a rational expectations equilibrium stated by Futia prevents rational agents from making use of the knowledge that the variables that they observe are the outcome of an equilibrium process. We argue that this is inconsistent with the standard definition of rational expectations. More precisely, in the "only if" direction of Futia's main existence argument, the agents act according to a *reduced-form* interpretation of the observed equilibrium variables; while in the "if" direction an argument is used that relies on a *structural* interpretation of the observed equilibrium variables on the part of the agents. The non-existence pathology originates from the "only if" part of the proof. Once the definition of the rational expectations equilibrium is modified to address such inconsistency, the non-existence pathology disappears. Where Futia (1981) thought an equilibrium did not exist, a Rational Expectations equilibrium identical to the Full Communication equilibrium does exist.

The broader message of this note is that in the presence of signal extraction from equilibrium variables, the fixed point of information that is part of any rational expectations characterization poses subtle challenges to the modeler, both at the definition and at the solution stages. However by resolving the existence pathologies of Futia, this paper pushes the literature forward by arguing that more information can be safely entrusted to equilibrium interactions, with the potential of gaining useful insights.

2. Futia (1981): framework and main results

Section 1 of Futia (1981) describes the model and is titled, "Land Speculation in Hilbert Space". He assumed that there is a fixed quantity of land with speculative and non-speculative traders. The non-speculative demand for land at each date t is a random variable (i.e., noise traders) that never exceeds the total supply. The difference between the total supply and non-speculative demand is denoted as s_t. The speculative demand of trader i for land arises from the demand function $q_t^i = \mathbb{E}_t^i p_{t+1} - \alpha p_t$, where $\alpha > 1$ is the opportunity cost of funds.[3]

Setting supply equal to demand delivers the forward looking linear expectational difference equation[4]

$$p_t = \beta \mathbb{E}\left[p_{t+1} | \Omega_t\right] + s_t \tag{2.1}$$

where $\alpha^{-1} = \beta < 1$, s_t is a covariance stationary process of the form $s_t = A(L)\varepsilon_t$ with $A(L)$ a square summable lag polynomial in non-negative powers of L and $\varepsilon_t \sim N(0, \sigma_\varepsilon)$ idiosyncratically distributed across time t. Expectations are assumed to be rational, which results in the expectation operator corresponding to linear projection upon the space defined by the information set; Ω_t is the information set available at time t of the representative agent. In this note, following Futia (1981) we restrict our attention to the case of symmetric information sets, i.e. a situation where all agents have access to the same, possibly incomplete, information. In terms of notation, we denote $\mathbb{V}_t(x)$ as the smallest closed linear subspace spanned by the infinite history of the random variable x_t up to time t, namely $x^t \equiv \{x_t, x_{t-1}, x_{t-2}, \ldots\}$. In what follows we will also assume that the polynomial $A(L)$ is invertible, so that observing the history of s_t is equivalent (in mean square norm) to observing the history of ε_t, formally $\mathbb{V}_t(s) = \mathbb{V}_t(\varepsilon)$.[5]

We next present the key steps in Futia (1981)'s existence results.

Full communication equilibrium

The first step consists of defining and solving for a Full Communication equilibrium (FC) as introduced by Radner (1979). It corresponds to solving the expectational equation under the assumption that the agents observe the history of innovations ε_t up to time t, i.e. $\Omega_t = \mathbb{V}_t(\varepsilon)$. The formal definition follows.

Definition FC. A full communication equilibrium is a stationary process for p_t such that for $t \in \mathbb{Z}$

1. $\mathbb{V}_t(p) \subseteq \mathbb{V}_t(\varepsilon)$.
2. $p_t = \beta \mathbb{E}\left[p_{t+1} | \mathbb{V}_t(\varepsilon)\right] + s_t$.

The following proposition states that in the present setting there always exists a unique FC equilibrium and provides a closed form solution.

Proposition 1. *A FC equilibrium exists, is unique, and is given by*

$$p_t = \left(\frac{LA(L) - \beta A(\beta)}{L - \beta}\right)\varepsilon_t. \tag{2.2}$$

Proof. See Appendix. □

Restating Futia (1981)

One of the main contributions of Futia (1981) is the argument that a relevant notion of a rational expectations equilibrium should take into account that agents observe variables that are themselves equilibrium outcomes, or, put differently, that the information set of agents arises endogenously. In the univariate context of the model (2.1) this corresponds to the assumption that the representative agent obtains information only from current and past equilibrium prices. Formally, the information set in (2.1) is specified as $\Omega_t = \mathbb{I}_t(p)$, to denote that the only source of information is transmitted by prices. Futia's definition of an equilibrium specifies a particular mathematical structure for $\mathbb{I}_t(p)$.

Definition FRE. A Futia rational expectations (FRE) equilibrium is a stationary process p_t such that for $t \in \mathbb{Z}$

1. $\mathbb{V}_t(p) \subseteq \mathbb{V}_t(\varepsilon)$.
2. $p_t = \beta \mathbb{E}\left[p_{t+1} | \mathbb{I}_t(p_t)\right] + s_t$ where $\mathbb{I}_t(p) = \mathbb{V}_t(p)$.

[2] See Rondina and Walker (2012a) for a systematic analysis of how the informational feedback in equilibrium can generate a new class of rational expectations equilibria.

[3] Rondina and Walker (2012a) derive the microfoundations that motivate the model of Futia (1981).

[4] Without loss of generality, we have suppressed the constant β multiplying s_t.

[5] Relaxing this assumption opens up a set of interesting implications for the dynamics of a rational expectations equilibrium (see Rondina and Walker (2012a)); for the purpose of this paper it is however not essential and so we abstract from it.