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Impacts of the initial observation on unit root tests using recursive demeaning and detrending procedures

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HIGHLIGHTS

- *Recursive* demeaning/detrending procedures have been very popular in the literature.
- The effect of the initial observation on recursive methods has not been addressed.
- The unit root tests using recursive methods lose power as the initial value gets large.
- The situation is similar to the DF-GLS type tests.

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ABSTRACT

The use of *recursive* demeaning and detrending procedures in unit root tests has been popular in the literature, since they lead to more precise estimation of the persistence parameter and greater power in unit root tests. However, we find that unit root tests using these recursive procedures tend to lose power significantly when the initial value is very large.

1. Introduction

This paper examines the effects of the initial observation on unit root tests using recursive demeaning and detrending procedures. *Recursive* demeaning and detrending procedures have been very popular in the literature. The pioneering work of So and Shin (1999) and Shin and So (2001) suggest that the usual practice of demeaning and detrending the data can lead to bias in the estimates of AR coefficients and have an adverse impact on unit root tests. They note that the source of the bias comes from possible correlation between y_{t+k} and e_t, $k > 0$. As such, a recursive procedure using data up to $t-1$ (y_j, $j = 1, \ldots, t-1$, rather than $j = 1, \ldots, T$) has been advocated to eliminate terms representing these correlations. Such treatment has proved very effective as the persistence parameter can be estimated more precisely and unit root tests using these recursive methods are more powerful. As a result, recursive methods have been widely utilized in the literature. For example, Cook (2002) and Kim and Moh (2010) employ a recursive demeaning procedure to correct for size distortions in unit root tests. Phillips et al. (2004) adopts recursive procedures in their nonlinear instrumental variable (NIV) estimations. Rodrigues (2006) examines the performance of various detrending methods. Such recursive methods are not restricted to unit root tests. Choi et al. (2010) employs the recursive mean adjustment procedure to reduce bias in estimating dynamic panel data models.

However, in spite of success from adopting the above recursive methods, an important question remains. What is the impact of the initial observation on various tests using these recursive

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methods? To our surprise, this issue has not been addressed in the literature. We examine this question in this paper. It is possible that the improved power noted in these tests might be obtained by inducing an unexpected consequence. In many statistical and econometric issues, it is typical to observe a trade-off situation where one solution method can come at the price of inducing other issues. In short, we find that a power loss problem can follow when recursive approaches are adopted. We find that this phenomenon occurs for both recursive demeaning and detrending procedures. Also, we find that the same issue exists in both OLS based and IV (instrumental variables) based unit root tests.

2. Recursive procedures

Suppose that one considers estimating an AR model with a non-zero mean

$$y_t - \mu = \beta(y_{t-1} - \mu) + e_t, \tag{1}$$

where μ is an unknown mean of y_t, and β is the AR coefficient. The discussion can be extended to a general ARMA(p, q) model, but we use a simple AR(1) model to illustrate the procedure. In order to control for the effect of the unknown mean, it is natural to use the sample means $\bar{y} = \frac{1}{T-1}\sum_{t=2}^{T} y_t$ and $\bar{y}_{-1} = \frac{1}{T-1}\sum_{t=2}^{T} y_{t-1}$. Then, the demeaned process, with $y_t^* = y_t - \bar{y}$ and $y_{t-1}^* = y_{t-1} - \bar{y}_{-1}$, is used to estimate the persistence parameter β.

$$y_t^* = \beta y_{t-1}^* + e_t. \tag{2}$$

It is known that the estimate of β involves a bias term, which is given by

$$E(\hat{\beta}) - \beta = -\frac{1}{T}(1 + 3\beta) + o(T^{-1}). \tag{3}$$

Then, under the null of a unit root, $\beta = 1$ in (1) with the initial observation $y_0 = 0$ and $\mathrm{var}(e_t) = \sigma^2$, Shin and So (2001) have shown that

$$E(y_{t-1} - \bar{y}_{-1})e_t = -\frac{1}{2}(T + 1)\sigma^2, \tag{4}$$

which contributes to the source of the bias in (3). The problem is on the power; it has been often claimed that the bias can yield loss of power in the usual unit root tests.

To mitigate the downward bias, So and Shin (1999) suggested using a recursive *demeaning* adjustment

(Method 1)

$$y_t^* = y_t - \bar{y}_{t-1}, \tag{5}$$

$$y_{t-1}^* = y_{t-1} - \bar{y}_{t-1}, \tag{6}$$

where $\bar{y}_{t-1} = \frac{1}{t-1}\sum_{k=1}^{t-1} y_k$. Then, the recursively demeaned regressor y_{t-1}^* will be independent of the error term e_t, since it does not involve the correlations between y_{t+k} and $e_t, k > 0$; $E\left(y_{t-1}^* e_t\right) = E\left(y_{t-1} - \bar{y}_{t-1}\right) e_t = 0$. In particular, So and Shin (1999) showed that the bias of $\hat{\beta}$ in (2) can be reduced to $E\left(\hat{\beta} - \beta\right) = O_p(T^{-1}\log T)$. Unit root tests based on the recursive demeaning procedure are invariant to any nuisance parameter under the null. However, in cases with a linear time trend the procedure is more complicated.

For the model with a linear time trend, Shin and So (2001) initially suggested the following recursive *detrending* procedure.

$$\tilde{y}_t = y_t - z_{t-1}'\tilde{\delta}_{t-1}, \tag{7}$$

$$\tilde{y}_{t-1} = y_{t-1} - z_{t-1}'\tilde{\delta}_{t-1}, \tag{8}$$

where $z_t = (1, t)'$, and $\tilde{\delta}_{t-1}$ is obtained recursively from OLS estimation of the regression of y_t on $z_t = (1, t)'$ using data up to $t - 1$

$$\tilde{\delta}_{t-1} = \left(\sum_{k=1}^{t-1} z_k z_k'\right)^{-1} \sum_{k=1}^{t-1} z_k y_k.$$

However, Rodrigues (2006) and Sul et al. (2005) note that unit root tests using the recursive detrending method in (7) and (8) depend on the nuisance parameter δ_1, which reflects the magnitude of the trend coefficient. Thus, the resulting tests are not pivotal under the null. Therefore, we do not examine the above procedure, but instead focus on a few alternative recursive detrending methods.

First, we examine the following recursive detrending procedure suggested by Chang and Park (2004).

(Method 2)

$$\tilde{y}_t = y_t - y_0 - \frac{1}{T}\sum_{t=1}^{T}\Delta y_t - \sum_{k=1}^{t-1}\frac{1}{k}\left(y_k - y_0\right), \tag{9}$$

$$\tilde{y}_{t-1} = y_{t-1} - y_0 - \sum_{k=1}^{t-1}\frac{1}{k}(y_k - y_0). \tag{10}$$

Second, we examine a modified procedure of Chang (2002).

(Method 3)

$$\tilde{y}_t = y_t + \frac{2}{t-1}\sum_{k=1}^{t-1} y_k - \frac{1}{T}\sum_{t=1}^{T}\Delta y_t - \frac{6}{t(t-1)}\sum_{k=1}^{t-1} k y_k, \tag{11}$$

$$\tilde{y}_{t-1} = y_{t-1} + \frac{2}{t-1}\sum_{k=1}^{t-1} y_k - \frac{6}{t(t-1)}\sum_{k=1}^{t-1} k y_k. \tag{12}$$

Finally, we examine the procedure of Taylor (2002), which may perform better in the OLS based tests, according to Rodrigues (2006).

(Method 4)

$$\tilde{y}_t = y_t + \frac{2}{t}\sum_{k=1}^{t} y_k - \frac{6}{t(t+1)}\sum_{k=1}^{t} k y_k, \tag{13}$$

$$\tilde{y}_{t-1} = y_{t-1} + \frac{2}{t-1}\sum_{k=1}^{t-1} y_k - \frac{6}{t(t-1)}\sum_{k=1}^{t-1} k y_k. \tag{14}$$

The point of these alternative procedures is to get rid of the dependency on the nuisance parameter. Method 4 is obtained from a recursively detrended y_t with data up to time t instead of $t - 1$, without subtracting the mean of Δy_t as is done in Methods 2 and 3. Using any of these recursive detrending methods, one may test the null of a unit root, $H_0 : \beta = 1$, using the testing equation (2).

3. Effects of the initial value

In this paper, we examine the impact of the initial observation on unit root tests using the recursive demeaning and detrending procedures described above. We consider an unobserved representation form of a time series

$$y_t = z_t'\delta + v_t \quad t = 1, \ldots, T, \tag{15}$$

$$v_t = \beta v_{t-1} + e_t, \tag{16}$$

where $z_t = 1$ for a model with a constant, or $z_t = (1, t)'$ for a model with a trend with $\delta = (\delta_0, \delta_1)'$, $v_0 = \xi$, and e_t follows a standard normal distribution, $e_t \sim N(0, 1)$.[1] To begin with, we allow the initial value ξ to take some fixed values with $\xi = 0, 5, 10, 20$ and

[1] Although we assume that e_t follows a standard normal distribution, using non-normal distributions, such as a chi-square or t-distribution, gives similar results.