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Asymmetric Nash bargaining solutions: A simple Nash program

Nejat Anbarci, Ching-jen Sun *

School of Accounting, Economics and Finance, Deakin University, 70 Elgar Road, Burwood, VIC 3125, Australia

HIGHLIGHTS

- A simple Nash program for asymmetric Nash solutions is proposed.
- We generalize the Nash demand game analyzed by Rubinstein et al. (1992).
- We provide an axiomatic characterization of the class of asymmetric Nash solutions.

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ABSTRACT

This article proposes a simple Nash program. Both our axiomatic characterization and our noncooperative procedure consider each distinct asymmetric and symmetric Nash solution. Our noncooperative procedure is a generalization of the simplest known sequential Nash demand game analyzed by Rubinstein et al. (1992). We then provide the simplest known axiomatic characterization of the class of asymmetric Nash solutions, in which we use only Nash's crucial Independence of Irrelevant Alternatives axiom and an asymmetric modification of the well-known Midpoint Domination axiom.

1. Introduction

In an important paper, Harsanyi and Selten (1972) proposed and axiomatically characterized the asymmetric (generalized) Nash solutions. Kalai (1977) provided a much simpler axiomatic characterization of these solutions by using three of the original axioms of Nash (1950), namely Weak Pareto Optimality (WPO), Scale and Origin Invariance (SOI) and Independence of Irrelevant alternatives (IIA).

In a variation of Nash demand game considered in Rubinstein et al. (1992), Player 1 makes a proposal s and Player 2 is free to accept 1's proposal, to continue negotiations even if he does not accept 1's proposal or to terminate negotiations. In case Player 2 rejects 1's proposal, 2 is more likely to continue negotiations if 1's proposal is more to his liking. Player 2 announces his probability $p \in [0, 1)$ to continue negotiations after he hears Player 1's proposal and rejects it (clearly Player 1 too can calculate Player 2's continuation probability p). If Player 2 continues negotiations, Player 1 will have to choose either the new proposal $\widetilde{s}$ made by Player 2 or to scale his original proposal down by p.

We first provide a simple generalization of the above game. Our generalization conceptually separates the latter scale-down scalar p and the continuation probability p in that they do not have to be equal. Our game, however, still keeps a link between them by making the former a function of the latter; each different link between them in our game gives rise to a distinct subgame-perfect equilibrium outcome which coincides with the outcome of a distinct asymmetric (or symmetric) Nash solution.

We then provide an axiomatic characterization of the class of asymmetric Nash solutions, in which we use only Nash's crucial Independence of Irrelevant Alternatives (IIA) axiom and an asymmetric modification of the well-known Midpoint Domination (MD) axiom.[1]

* Corresponding author.
E-mail addresses: nejat.anbarci@deakin.edu.au (N. Anbarci), econcjs@gmail.com, cjsun@deakin.edu.au (C.-j. Sun).

[1] MD was proposed by Sobel (1981) and Moulin (1983) provided the simplest axiomatic characterization of the (symmetric) Nash solution by using MD and IIA only.

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Ours is a very simple Nash program[2] because it provides (i) the simplest known axiomatic characterization of the asymmetric (generalized) Nash solutions axioms, and (ii) a noncooperative procedure that is a generalization of the simplest sequential Nash demand game analyzed by Rubinstein et al. (1992).

2. Nash demand game(s) and the Nash solution(s)

Nash (1950) provided the first axiomatic characterization of a cooperative bargaining solution. Nash (1953) provided the first noncooperative justification of his solution concept by using his own (Nash) demand game (NDG). In that game, two players simultaneously make demands; each player receives the payoff he/she demands if the demands are jointly feasible, and nothing otherwise. NDG has a major downside, however: every point on the Pareto frontier is a Nash equilibrium outcome. Nash (1953) himself tried to rectify this problem by utilizing a "smoothing" approach in which with some positive probability, incompatible demand combinations did not lead to zero payoffs. This smoothing approach uniquely provided non-cooperative foundations for the Nash solution as the above-mentioned probability tends to zero; however, it was not deemed reasonable by game theorists and several alternatives have been proposed (Luce and Raiffa, 1957; Schelling, 1960).

In Carlsson (1991), the set of feasible payoffs is known to both players, but their actions are subject to some errors; in addition, unlike in the NDG, if players make demands which do not exhaust the available surplus, the remainder is distributed according to an exogenously fixed rule. In the limit as the noise vanishes, the equilibrium outcome converges to one of the asymmetric Nash solution outcomes. The rule about the proportion of the unclaimed surplus that is supposed to go to each of the players determines which particular asymmetric Nash solution outcome will be obtained.

Howard (1992) proposed a one-shot (multiple-stage) noncooperative foundation for the (symmetric) Nash solution, which was later significantly simplified by Rubinstein et al. (1992), which was alluded to in the Introduction briefly and will be discussed in more detail later.

Binmore et al. (1986) showed that as the time between alternating offers by players in the Rubinstein (1982) bargaining game tends to zero, the unique subgame perfect equilibrium outcome corresponds to one of the asymmetric Nash solutions, depending on the relative discount factors of the players. Kultti and Vartiainen (2010) generalize Binmore et al. (1986); they show that differentiability of the payoff set's Pareto frontier is essential for the convergence result if there are at least three players.

3. Asymmetric Nash solutions: a simple Nash program

A two-person *bargaining problem* is a pair (S, d), where $S \subset \mathbb{R}^2$ is the *set of utility possibilities*, and $d \in S$ is the *disagreement point*, which is the utility allocation that results if no agreement is reached by the two parties. It is assumed that (1) S is compact and convex, and (2) $x > d$ for some $x \in S$.[3] Let Σ be the class of all two-person problems satisfying (1) and (2) above. Define $IR(S, d) \equiv \{x \in S | x \geq d\}$ and $WPO(S) \equiv \{x \in S | \forall x' \in \mathbb{R}^2$, with $x' > x \Rightarrow x' \notin S\}$. A *solution* is a function $f : \Sigma \to \mathbb{R}^2$ such that for all $(S, d) \in \Sigma, f(S, d) \in S$. The asymmetric Nash solution with weight $\alpha \in (0, 1)$, N^α, selects $N^\alpha(S, d) = \arg\max\{(x_1 - d_1)^\alpha (x_2 - d_2)^{1-\alpha} | x \in IR(S, d)\}$ for each $(S, d) \in \Sigma$.[4]

For simplicity in our noncooperative analysis let us normalize d such that $d = (0, 0)$.

Consider the following Nash demand game proposed by Rubinstein et al. (1992):

Stage 1. Player 1 proposes a division $s \in S$.

Stage 2. Player 2 proposes an alternative division $\widetilde{s} \in S$ and a probability $p \in [0, 1]$.

Stage 3. The game continues with probability p and terminates at $(0, 0)$ with probability $1 - p$.

Stage 4. Player 1 chooses between $\widetilde{s}$ and ps.

In the Introduction, we gave an intuitive description of this game. We can add the following explanation of how the equilibrium is obtained. Observe that at Stage 4, ps and $\widetilde{s}$ depend on 1's initial proposal s. Player 2 will reciprocate with a higher continuation probability p and with a more favorable ps as well as a more favorable counter-offer $\widetilde{s}$ for Player 1, if 1's initial proposal is more favorable for 2. On the other hand, the less s is to 2's liking, the lower ps and the less favorable $\widetilde{s}$ are for Player 1. Thus, if at Stage 1 s is less to 2's liking, at Stage 2 Player 2 will continue negotiations with a lower probability p and force Player 1 to choose between worse new alternatives $\widetilde{s}$ and ps at Stage 4.

In turn, Player 1 can avoid all of this and obtain Player 2's immediate acceptance of s if s is above some particular threshold. The setup is symmetric. This particular threshold for s would be the same if players reversed roles, i.e., if Player 2 instead of Player 1 started the procedure by proposing s.

Observe that by proposing a low p, Player 2 is potentially punishing himself as well since the game will continue with a lower probability. But once Player 2 continues negotiations, the punishment by the scalar p pertains primarily to Player 1 which will make Player 1 settle for $\widetilde{s}$ instead. But since the continuation probability p at Stage 2 and the scalar p at Stage 4 are the same, Player 2 must pick a lower continuation probability p in order to name a lower scalar p to punish primarily Player 1 (who does not care about the scaling down of Player 2's payoff s_2 in s, but only cares about the scaling down of his own payoff s_1 in s).

Note that in the above game by Rubinstein et al. (1992) there is no reason why the continuation probability p proposed by Player 2 as well as the fraction p of s proposed by Player 2, that Player 1 has to choose against $\widetilde{s}$ in Stage 4 have to be equal. As a matter of fact, these two p's pertain to two totally different things. The former is the probability with which Player 2 will continue negotiations, and the latter is a proposed fraction of the payoff s by 2.

Consider the following variation of the above Nash demand game, which deliberately separates the continuation probability p and the fraction of payoff s (while still keeping a link between them):

Stage 1. Player 1 proposes a division $s \in S$.

Stage 2. Player 2 proposes an alternative division $\widetilde{s} \in S$ and a probability $p \in [0, 1]$.

Stage 3. The game continues with probability p and terminates at $(0, 0)$ with probability $1 - p$.

Stage 4. Player 1 chooses between $\widetilde{s}$ and qs, where $q = p^\theta$ and $\theta \in (0, \infty)$.

We call this θ-weighted Nash demand game. Note that when $q = p$, then this game boils down to the original Rubinstein et al. game. The use of this more general form for q in Stage 4 suggests that, if there is a need to link q and p, then this link need not always be in the strict form of $q = p$.

[2] The Nash program attempts to bridge the gap between the cooperative (axiomatic) and non-cooperative (strategic) strands of game theory by providing non-cooperative procedures that yield cooperative solutions' outcomes as their equilibrium outcomes. See Serrano (2008) more on the Nash program.

[3] Given $x, y \in \mathbb{R}^2$, $x > y$ if $x_i > y_i$ for each i, and $x \geq y$ if $x_i \geq y_i$ for each i and $x_i > y_i$ for some i.

[4] Observe that each $N^\alpha(S, d)$ is strongly Pareto optimal.