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Derivation of marginal effects of determinants of technical inefficiency

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HIGHLIGHTS

- We model the environmental variables such that the mean and variance of error term are functions of them.
- We derive the marginal effects of an environmental variable on the JLMS estimator.
- We apply the modeling framework and formula of marginal effects to a banking data set.

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ABSTRACT

In efficiency studies using the stochastic frontier approach, the main focus is to explain inefficiency in terms of some exogenous variables and computation of marginal effects of each of these determinants. Although inefficiency is estimated by its mean conditional on the composed error term (the Jondrow et al., 1982 estimator), the marginal effects are computed from the unconditional mean of inefficiency (Wang, 2002). In this paper we derive the marginal effects based on the Jondrow et al. estimator and use the bootstrap method to compute confidence intervals of the marginal effects.

1. Introduction

In stochastic frontier models the two main objectives are to estimate underlying production technology and observation-specific technical inefficiency. While estimating inefficiency, the empirical studies in this literature examine whether differences in inefficiency among producers can be explained in terms of some exogenous (environmental) variables. A natural question in this context is how to compute the marginal effects of these environmental variables on inefficiency. For this, first we need a model that includes these environmental variables in the specification of inefficiency, and then a point estimator of inefficiency. Models in which the environmental variables enter into the mean and/or the variance of inefficiency have been proposed in some earlier studies, e.g., Kumbhakar et al. (1991), Reifschneider and Stevenson (1991), Huang and Liu (1994), and Battese and Coelli (1995).[1] Wang (2002) examined this model thoroughly by allowing the environmental variables to enter into the mean and the variance of inefficiency and derived the formula for calculating marginal effects of the environmental variables on inefficiency. However, his derivation of the marginal effects was based on the unconditional mean of inefficiency, although the estimator of inefficiency was based on the conditional mean – the Jondrow et al. (1982) (henceforth JLMS) estimator.

In this paper we derive the marginal effects of environmental variables on inefficiency where inefficiency is estimated using the Jondrow et al. (1982) formula. We consider a model in which the environmental variables appear in both the mean and the variance of inefficiency as well as in the variance of the noise term. Based on this model we show that there are three channels through which the environmental variables can effect the estimated inefficiency. We show that even if these variables do not enter into either the mean or the variance of inefficiency, they can affect inefficiency via the variance of the noise component. This is a new result which comes from the fact that the JLMS estimator (shown later) depends on the variance of the noise component. Since the JLMS estimator is universally used for estimating inefficiency, it should also be used to compute the marginal effects. That is, both inefficiency and its marginal effects should be based on the same formula.

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[1] Although chronologically last, in the efficiency literature these models are known as the Battese–Coelli (1995) model.

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The variance of the noise component in our model is made a function of the same environmental variables that affect the mean and/or the variance of inefficiency. Since we use the JLMS estimator which (as shown later) is a function of the mean and the variance of inefficiency as well as the variance of the noise component, the marginal effects of environmental variables will have a component coming from the variance of the noise term. This extra component was absent from the Wang (2002) formula and has not been discussed in any studies before because the JLMS estimator was not used in the literature to compute marginal effects.[2]

We apply European banking data and estimate inefficiency as well as the marginal effects. We compare our marginal effects with those based on Wang (2002). We also compute confidence intervals of the marginal effects using bootstrap procedure. Our results show that the marginal effects based on the Wang (2002) formula tend to overestimate marginal effects in our application.

2. A stochastic frontier model with environmental variables

Consider a stochastic production frontier model in a cross-sectional setting, viz.,

$$y_i = \beta' x_i + v_i - u_i, \tag{1}$$

$$u_i \sim N^+(\mu_i, \sigma_{ui}^2), \tag{2}$$

$$v_i \sim N(0, \sigma_{vi}^2), \tag{3}$$

$$\mu_i = c_0 + \delta' z_i, \tag{4}$$

$$\sigma_{ui} = \exp(c_1 + \gamma' z_i), \quad \text{and} \tag{5}$$

$$\sigma_{vi} = \exp(c_2 + \rho' z_i), \tag{6}$$

where u_i is the non-negative technical inefficiency component, which follows a truncated normal distribution. The vector of environmental variables z_i are allowed to affect the pre-truncation mean and variance of u_i, μ_i and σ_{ui}^2, respectively. The noise component is v_i distributed normally with zero mean and variance σ_{vi}^2 which is assumed to be a function of z_i as well.[3] Following Jondrow et al. (1982), it can be shown that the distribution of u_i given the composed error $\varepsilon_i = v_i - u_i$ is truncated normal with mean $\tilde{\mu}_i = (\mu_i \sigma_{vi}^2 - \varepsilon_i \sigma_{ui}^2)/\sigma_i^2$ and standard deviation $\sigma_{*i} = \sigma_{ui}\sigma_{vi}/\sigma_i$, where $\sigma_i^2 = \sigma_{ui}^2 + \sigma_{vi}^2$. Thus the point estimator of u_i is given by the conditional mean, i.e.,

$$E(u_i|\varepsilon_i) = \tilde{\mu}_i + \sigma_{*i} \frac{\phi(\tilde{\mu}_i/\sigma_{*i})}{\Phi(\tilde{\mu}_i/\sigma_{*i})}, \tag{7}$$

where ϕ and Φ denote the standard normal density and distribution functions, respectively. The estimator in (7) is known as the JLMS estimator in the efficiency literature.

Wang (2002) used the formula in (7) to calculate inefficiency but he used the post-truncation mean of u_i, i.e., $E(u_i|u_i > 0) = \sigma_{ui}\left[\Lambda_i + \frac{\phi(\Lambda_i)}{\Phi(\Lambda_i)}\right]$, where $\Lambda_i = \mu_i/\sigma_{ui}$, to compute the marginal effects. In other words, his marginal effects are computed from $\frac{\partial E(u_i|u_i>0)}{\partial z_{li}}$, where z_{li} is the l-th element of z_i. More specifically, the formula for the marginal effects in Wang (2002) is:

$$\frac{\partial E(u_i)}{\partial z_{li}} = \delta_l \left\{ 1 - \Lambda_i \frac{\phi(\Lambda_i)}{\Phi(\Lambda_i)} - \left[\frac{\phi(\Lambda_i)}{\Phi(\Lambda_i)}\right]^2 \right\} + \gamma_l \sigma_{ui} \left\{ (1+\Lambda_i^2)\frac{\phi(\Lambda_i)}{\Phi(\Lambda_i)} + \Lambda_i \left[\frac{\phi(\Lambda_i)}{\Phi(\Lambda_i)}\right]^2 \right\}, \tag{8}$$

where δ_l and γ_l are the coefficients associated with z_{li} in (4) and (5), respectively.

To derive the formula for the marginal effects based on the JLMS estimator in (7), i.e., $\frac{\partial E(u_i|\varepsilon_i)}{\partial z_{li}}$, we define $m_i = \tilde{\mu}_i/\sigma_{*i}$ and $g_i = \phi(m_i)/\Phi(m_i)$. After a lengthy and tedious algebra (which are skipped here but available from the authors upon request) the marginal effects of the l-th environmental variable on $E(u_i|\varepsilon_i)$ is found to be:

$$\begin{aligned}\frac{\partial E(u_i|\varepsilon_i)}{\partial z_{li}} &= \delta_l \left[\frac{\sigma_{vi}^2}{\sigma_i^2}(1 - m_i g_i - g_i^2)\right] \\ &\quad + \gamma_l \frac{1}{\sigma_i^2}\left\{\sigma_{vi}^2 \sigma_{*i}\left[g_i(1+m_i^2) + m_i g_i^2\right] \right. \\ &\quad \left. -\ 2\sigma_{*i}^2(\varepsilon_i + \mu_i)(1 - g_i^2 - m_i g_i)\right\} \\ &\quad + \rho_l \frac{1}{\sigma_i^2}\left\{\sigma_{ui}^2 \sigma_{*i}\left[g_i(1+m_i^2) + m_i g_i^2\right] \right. \\ &\quad \left. + 2\sigma_{*i}^2(\varepsilon_i + \mu_i)(1 - g_i^2 - m_i g_i)\right\},\end{aligned} \tag{9}$$

where δ_l, γ_l and ρ_l are the coefficients associated with z_{li} in (4), (5) and (6), respectively. This result shows that the marginal effects of z_l have three components which identify three separate channels through which z_l affects the estimated inefficiency. These components/channels are related to μ_i, σ_{ui} and σ_{vi} functions. That is, if μ_i, σ_{ui} and σ_{vi} are functions of z_i, then each element of z_i affects inefficiency via the three channels given by the three terms on the right-hand-side of (9). On the other hand, if σ_{ui} and σ_{vi} are constants, then the marginal effects come only from the mean and it is δ_l multiplied by an adjustment function which is positive.[4] Similarly, if μ_i is constant (i.e., u_i follows a truncated-normal distribution with heteroskedasticity) the first term in (9) drops out and the channels by which z_l affects inefficiency are through the variances of u_i and v_i. The same holds true when $\mu_i = 0$ (i.e., u_i follows a half-normal distribution with heteroskedasticity). If $\mu_i = 0$ and σ_{vi}^2 is a constant, then z_l affects inefficiency through σ_{ui}^2 and this is captured by the second term in (9). Finally, the new result in (9) comes from the last term that captures the effect via σ_{vi}^2. This term is new and is not explored in the literature.[5] This component is interesting because it shows that if σ_{vi}^2 is a function of z_i, these variables can also affect inefficiency even if $\mu_i = 0$ and σ_{ui}^2 is a constant.

3. Empirical application

As an empirical illustration, we use an unbalanced panel data of European banking industry obtained from Bankscope.[6] The data set covers 15 European countries[7] for 17 years (during the period between 1993 and 2009) with a total of 6733 bank-year observations. Although we considered a cross-sectional model and our derivation of marginal effects is based on that cross-sectional model, the panel extension of it is trivial. For this we add an extra subscript t to z_i, μ_i, σ_{ui}^2 and σ_{vi}^2, and specify the model

[2] Wang (2002) used the unconditional mean of inefficiency (which is independent of the variance of the noise term) to compute the marginal effects and this is why the extra term was missing in his derivation.

[3] Note that $\sigma_{ui}^2 = \exp(2c_1 + 2\gamma' z_i)$ and $\sigma_{vi}^2 = \exp(2c_2 + 2\rho' z_i)$.

[4] Note that this function is similar to the one in (8) (Eq. 9 in Wang (2002), p. 244), except for the extra term of σ_{vi}^2/σ_i^2.

[5] It can be seen from (8) that the Wang (2002) formula did not have this component although he allowed the variance of the noise component, σ_{vi}^2, to depend on z variables (in the application part). This is because $E(u_i|u_i > 0)$ does not depend on σ_{vi}^2.

[6] See www.bankscope2.bvdep.com for details.

[7] These are Austria, Belgium, Denmark, Finland, France, Germany, Great Britain, Greece, Ireland, Italy, Luxembourg, The Netherlands, Portugal, Spain, and Sweden.