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## Skill distribution and the optimal marginal income tax rate

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#### 1. Introduction

Mirrlees (1971) developed a seminal framework to study nonlinear income taxation, particularly the behavior of the optimal marginal tax rate, and showed that the marginal tax rate should be nonnegative and should be zero for the highest skilled individual. Tuomala's simulations showed the optimal marginal tax rates can be inverse U-shaped (Tuomala, 1990). Myles (2000) simulations indicated that, except for the fact that it is zero for the highest skilled individual, the optimal marginal tax rates in the interior of the skill distribution can be of any shape depending on skill distribution. Diamond (1998) found that the optimal marginal tax rates follow a U-shaped pattern with the utility function being quasi-linear in consumption and special skill distributions. Using a logarithmic utility function, Dahan and Strawczynski (2000) showed that optimal income tax rates can be inverse U-shaped by replacing quasi-linearity in consumption. They concluded that the form of utility function is solely responsible for the shift from upward to downward sloping of the optimal tax curve at high levels of income.

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ABSTRACT

This paper emphasizes the role of skill distribution in determining the optimal marginal tax rates. It rigorously shows that the optimal marginal tax rates can be strictly increasing, strictly decreasing, U-shaped, or inverse U-shaped as skill increases depending on the skill distribution.

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With the same utility function and social welfare function utilized by Diamond (1998), but with different density functions of skill distribution, this paper makes a contribution to the literature that shows why there can be no general result on the structure of the optimal marginal tax rate. We demonstrate rigorously that the optimal marginal tax rate can be strictly increasing, strictly decreasing, U-shaped, or inverse U-shaped, depending on the distribution of skills. We suggest some features of the distribution that give rise to particular patterns of the marginal rate.

#### 2. The model

Let *n* be the skill or wage for some individuals in this economy. Assume that in this economy, the skill level *n* has a low boundary  $n_0$  and an upper boundary  $n_1$  satisfying  $0 \le n_0 < n_1$ . Let *x* and *y* denote consumption and labor of someone with skill *n*, respectively (*x* and *y* are functions of *n*). Assume that the utility function is u(x, y) = x + v(1 - y), where *v* is assumed to be a strictly increasing, strictly concave and twice continuously differentiable function, and *y* is the percentage of time spent on work ( $0 \le y \le 1$ ). Individuals with skill level *n* choose a work time y(n) and consumption x(n) to maximize *u*. Let x(n) and y(n) denote the optimal values of consumption *x* and labor supply *y*, i.e.,  $u(x(n), y(n)) = \max_{x,y} u(x, y)$ . Let F(n) be the distribution of skills with density f(n).



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The social welfare function is as follows:

$$\int_{n_0}^{n_1} G[u(x(n), y(n))]f(n)dn$$
(1)

where *G* is a strictly concave function. The government collects taxes, T(ny(n)), from an individual with income ny(n). Let *E* be government expenditures. Since taxes are the difference between income and consumption, the government budget constraint is:

$$\int_{n_0}^{n_1} T[ny(n)]f(n)dn = \int_{n_0}^{n_1} ny(n)f(n)dn - \int_{n_0}^{n_1} x(n)f(n)dn \ge E.$$
 (2)

Since an individual's consumption is x(n) = ny - T(ny), the utility function becomes a single variable function of y, u(x, y) = ny - T(ny) + v(1 - y). Maximizing u by only choosing y yields:

$$v'[1 - y(n)] = n\{1 - T'[ny(n)]\}v'(1 - y) = n[1 - T'(ny)].$$
 (3)

The incentive compatibility constraint is:

$$U[ny(n) - T(ny(n)), y(n)] \\
\geq u[n'y(n') - T(n'y(n')), n'y(n')/n]$$
(4)

for all *n* and *n'* where  $n \in [n_0, n_1]$  and  $n' \in [n_0, n_1]$ .

The optimal income taxation problem is to choose labor supply, *y*, to maximize the social welfare in Eq. (1) subject to Eqs. (2)–(4). Let e(n) be defined as follows:

$$e(n) = -\frac{v'[1 - y(n)]}{y(n)v''[1 - y(n)]}.$$
(5)

We denote  $e^{-1} = 1/e(n)$ . Diamond (1998) solved this problem and obtained the following equation to characterize the optimal tax rate for an individual with skill *n*:

$$\frac{T'}{1-T'} = \left(e^{-1}+1\right) \frac{\int_n^{n_1} (p-G') dF}{pnf(n)}$$
$$= \frac{e^{-1}+1}{n} \frac{\int_n^{n_1} (p-G') dF}{p[1-F(n)]} \frac{1-F(n)}{f(n)}$$
(6)

where p is the Lagrange multiplier on the government budget constraint, G'[u(n)] is marginal social welfare of an individual's utility, and

$$\int_{n_0}^{n_1} (p - G') dF = 0.$$
 (7)

Since G''(u) < 0, p - G' is strictly increasing in u, it is easy to show that u(n) is increasing in n, and thus, G''(u)u'(n) < 0 and p - G' is also strictly increasing in n, i.e., d(p - G')/dn > 0. The following properties hold:

$$p - G'(n) < 0 \text{ if } n < n_c, p - G'(n_c) = 0 \text{ if } n = n_c, \text{ and } (8) p - G'(n) > 0 \text{ if } n > n_c.$$

Let G'[u(x(n), y(n))] = G'[u(n)]. There exists  $n_c \in (n_0, n_1)$  satisfying

$$G'[u(n_c)] = G'\{u[x(n_c), y(n_c)]\} = \int_{n_0}^{n_1} G' dF.$$
(9)

Diamond (1998) showed that if the utility of leisure satisfies  $v(1 - y) = c(1 - y^k)$ , for some constant c > 0 and k > 1, then the elasticity of labor supply is constant  $e^{-1} + 1 = k$ , for all *n*. The

assumption on utility function is crucial and it makes the analysis much simpler. Let

$$D(n) = \int_n^{n_1} (p - G') dF / nf(n).$$

We assume that  $v(1 - y) = c(1 - y^k)$ . From the above notation, Eq. (6) becomes

$$\frac{T'}{1-T'} = \frac{k}{p}D(n).$$
 (10)

In the case  $0 \le T' < 1$ , marginal tax rate T' is increasing if and only if T'/(1-T') is increasing, or equivalently, D(n) is increasing.

#### 3. Income distribution and optimal marginal tax rate

This section provides examples to show that the optimal marginal tax rate depends on income (skill) distribution.

**Proposition 1.** If  $u(x, y) = x + c(1 - y^k)$ , for some c > 0, and k > 1, and the skill density function is  $f(n) = \frac{\lambda}{n^{1+\alpha}}$ ,  $n_0 \le n < \infty$  with  $\alpha > 0$ ,  $\lambda > 0$ ,  $0 < n_0 < \infty$ , then the optimal marginal tax rates are increasing on  $[n_0, \infty)$ .

**Proof.** If  $u(x, y) = x + c(1 - y^k)$ , for some c > 0, and k > 1, and  $f(n) = \frac{\lambda}{n^{1+\alpha}}$ ,  $n_0 \le n < \infty$ , then

$$D(n) = \frac{\int_{n}^{\infty} \{p - G'[u(t)]\} \frac{\lambda}{t^{1+\alpha}} dt}{n\lambda/n^{1+\alpha}} = \frac{\int_{n}^{\infty} \{p - G'[u(t)]\} t^{-1-\alpha} dt}{n^{-\alpha}} \text{ and}$$
$$D'(n) = \frac{-\{p - G'[u(n)]\} n^{-1-\alpha} n^{-\alpha} + \alpha \int_{n}^{\infty} \{p - G'[u(t)]\} t^{-1-\alpha} dt n^{-1-\alpha}}{n^{-2\alpha}}.$$

Since p - G' is strictly increasing as *n* increases, we have the following inequality based on the basic property of a definite integral, which is crucial in proving this and the remaining propositions:

$$\alpha \int_{n}^{\infty} \{p - G'[u(t)]\} t^{-1-\alpha} dt n^{-1-\alpha}$$
$$> \alpha \{p - G'[u(n)]\} \int_{n}^{\infty} t^{-1-\alpha} dt n^{-1-\alpha}.$$

Thus,

$$\begin{split} D'(n) > & \frac{-\{p - G'[u(n)]\}n^{-1-\alpha}n^{-\alpha} + \alpha\{p - G'[u(n)]\}\int_{n}^{\infty}t^{-1-\alpha}dtn^{-1-\alpha}}{n^{-2\alpha}} \\ & = \frac{-\{p - G'[u(n)]\}n^{-1-2\alpha} + \alpha\{p - G'[u(n)]\}\frac{1}{-\alpha}(-n^{-\alpha})n^{-1-\alpha}}{n^{-2\alpha}} \\ & = 0. \end{split}$$

Since k > 0 and p > 0, we have  $\frac{d(T'/(1-T'))}{dn} = \frac{k}{p}D'(n) > 0$  and  $\frac{dT'}{dn} > 0$ .  $\Box$ 

In a special case where  $n_0 = 0.5$  and  $\alpha = 1$ , we have  $\lambda = 0.5$ , and  $f(n) = 0.5/n^2$ ,  $0.5 \le n < \infty$ . The probability density function is decreasing (see Fig. 1(a)).

The next proposition shows that the optimal marginal tax rate is strictly decreasing.

**Proposition 2.** If  $u(x, y) = x + c(1 - y^k)$ , for some c > 0, and k > 1, and the skill density function is  $f(n) = \lambda n^{\alpha}$ ,  $0 \le n \le n_1$ , where  $\alpha > -1$ ,  $\lambda > 0$ ,  $n_1 < \infty$ , then the optimal marginal tax rates are decreasing on  $(0, n_1]$ .

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