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A model of leverage based on risk sharing

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ABSTRACT

This paper offers a new approach, based on risk sharing, to endogenize the leverage of financial intermediaries. It endogenizes debt as the optimal contract for external financing, thereby capturing two features of leverage: debt serves to boost the return on equity, and equity provides "safety net" for debt. The paper derives a novel prediction that when the asset-side risk rises, the leverage ratio is reduced, but the profit margin of leveraging is actually widened.

1. Introduction

Leverage is ubiquitously used in the financial sector for boosting investment performance. For example, a study by British Private Equity & Venture Capital Association and Ernest & Young finds that half of the average return of the 14 biggest private equity deals realized in 2005–7 "came from the use of extra debt…".[1] This paper offers a new approach to study how financial intermediaries, such as banks and private equity funds, decide on leverage when seeking to boost their investment performance.

The approach here is based on risk sharing and can be illustrated as follows. Suppose a risk neutral bank and many extremely risk averse households each have \$1 to invest. There are two investment channels. One is risk free with gross return rate 1. The other is risky with a gross return rate of 0.9 or 1.3, each with probability one half, so the expected rate is 1.1. The bank, being risk neutral, puts its dollar in the risky asset earning an expected revenue of \$1.1, which gives a modest 10% net return rate. However, the bank can do better by taking in households' funds. Suppose it takes in one household's dollar and invests the entire \$2 (the taken-in one plus its own one) in the risky asset. The investment returns $\$2 \times 0.9 = \$1.8$ in the bad state and $\$2 \times 1.3 = \$2.6$ in the good state. The household is satisfied with getting \$1 back in both states. The bank thus earns, with this minor leverage, an expected revenue of $0.5 \times \$(1.8 - 1) + 0.5 \times \$(2.6 - 1) = \$1.2$, \$0.1 more than it earns without leverage. This extra \$0.1 is the difference between \$1.1, the return on one dollar investment on the asset side, and \$1, the repayment on the liability side to the household. Note that this repayment satisfies the household because it is risk free, which in turn is because his claim is senior to the bank's. That is, the contract to the household is debt and the bank's dollar forms the equity acting as the cushion to absorb loss to the household. The optimal amount of borrowing is \$9. At this level, the bad state revenue, $\$(1 + 9) \times 0.9 = \$9$, exactly suffices to service the risk-free debt. With this leverage, the bank earns \$2 for its dollar and achieves a shining net return rate of 100%. Thus the approach captures how leverage can be used to boost the performance of equity (i.e. the bank's dollar). Moreover, it explains why debt is cheap, namely, why the debt holders are willing to accept a return rate lower than prevails on the asset side, enabling debt to be used to raise the rate of return on equity.

This approach could be regarded dual to that used by Gale (2003, 2004) and Gale and Ozgur (2005). In the present paper, the risk averse agents get the reserved utility (which is endogenized) and the profit of the risk neutral agents is maximized, while in their papers, the risk neutral agents get their reservation profit and the utility of those risk averse is maximized. With this innovation, the approach here captures how leverage is used to boost the performance of equity investment.

There are other approaches to leverage decisions. The leverage ratio is determined by a Value-at-Risk rule in, e.g., Brunnermeier and Pedersen (2009) and Liu and Mello (2008), or by non-default

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[1] See "Buy-out Profits Tied to Debt", *Financial Times*, January 15, 2009.

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requirement in, e.g., Geanakoplos and Polemarchakis (1986), Geanakoplos (1997), Fostel and Geanakoplos (2008), or driven by risk-shifting concerns in, e.g., Adrian and Shin (2008). Compared to these approaches, the approach here endogenizes debt as the optimal contract for external financing and explains why it is cheap.

Moreover, in contrast with all the above literature, the paper derives a novel prediction that when the asset-side risk rises, the profit margin from leveraging is actually widened, though the leverage ratio is reduced.

The rest of the paper is organized as follows. Section 2 sets up the model and then analyzes it. Section 3 concludes. All the proofs are relegated to Appendix.

2. The model

There are a continuum of 1 unit banks and that of N units of households. Banks are risk neutral and protected by limited liability. For $i \in [0, 1]$, bank i has \$$K_i$ of funds. Let $K \equiv \int_0^1 K_i$ denote the aggregate supply of banks' funds. Households are risk averse, their preferences represented by a strictly concave function $U(\cdot)$, with $U'(\cdot) > 0$ and $U''(\cdot) < 0$, and each has a small amount of funds to invest, normalized to \$1.

Funds are either put into 1-to-1 storage or invested in a risky asset, of which the gross rate of return is $\overline{R}$ with probability q and $\underline{R}$ with probability $1 - q$, where $\overline{R} > \underline{R}$. Assume

$$R_e \equiv q\overline{R} + (1 - q)\underline{R} > 1 \tag{1}$$

$$qU(\overline{R}) + (1 - q)U(\underline{R}) < U(1). \tag{2}$$

Furthermore, to simplify the exposition, I assume that households cannot split their dollars between two channels of investment.

By (1), banks invest all their funds in the risky asset and earn a return rate of R_e. However, as was illustrated in Section 1, banks can boost the performance of investment by borrowing from households. Let L_i denote the amount of bank i's borrowing and $L \equiv \int_0^1 L_i$ denote the aggregate demand of households' funds by banks. The market where banks compete for households' funds, by the non-split assumption, is cleared by a *certainty equivalent return rate*, denoted by r, such that a household gets payoff $U(r)$ if investing in a bank rather than storing its dollar. The competitive equilibrium is thus defined as follows.

Definition 1. A profile of $(r; \{L_i\}_{i\in[0,1]})$ forms a competitive equilibrium

(i) given r, bank i takes in \$$L_i$ from the households;
(ii) if $r > 1$, then $L = N$; and if $0 < L < N$, then $r = 1$.

Condition (i) is self-evident. Condition (ii) presents how the capital market is cleared: if $r > 1$, then all households prefer investing in banks to storage and thus the market is cleared at $L = N$; if only part of households invest in banks, namely, $0 < L < N$, then households must be indifferent between such investment and storage, that is, $r = 1$.

Below I first examine how a typical bank decides on its leverage (i.e. condition i), then moving on to market clearing (i.e. condition ii).

2.1. The leverage of the representative bank

Suppose bank i, which has \$$K_i$ of its own funds, takes in L_i households' funds by issuing a security that promises to repay each of them $\overline{h}$ in the good state (when $\overline{R}$ is realized) and $\underline{h}$ in the bad state (when $\underline{R}$ is realized). Let $h_e \equiv q\overline{h} + (1 - q)\underline{h}$. The bank invests all $K_i + L_i$ of funds in the risky asset, which returns $(K_i + L_i)\overline{R}$ in the good state and $(K_i + L_i)\underline{R}$ in the bad one.[2] The total liability repayment is $L_i\overline{h}$ and $L_i\underline{h}$ respectively. Hence, the bank's expected profit is $\Pi = q((K_i + L_i)\overline{R} - L_i\overline{h}) + (1 - q)((K_i + L_i)\underline{R} - L_i\underline{h})$.

The problem of the bank is thus to find $\{L_i, \overline{h}, \underline{h}\}$ that maximizes Π subject to the following individual rationality (IR) and limited liability constraints.

Firstly, security $(\overline{h}, \underline{h})$ must give the investing households payoff $U(r)$ which they could get by investing in other banks (or from storage if $r = 1$), namely

$$qU(\overline{h}) + (1 - q)U(\underline{h}) = U(r). \tag{3}$$

Secondly, in both states, the investment revenue suffices to cover the liability outlay:

$$L_i\overline{h} \le (K_i + L_i)\overline{R} \tag{4}$$

$$L_i\underline{h} \le (K_i + L_i)\underline{R}. \tag{5}$$

The solution of the problem is characterized as follows.

Proposition 1. (i) *If $r > R_e$, then $L_i = 0$.*
(ii) *If $r = R_e$, then the bank is indifferent with any $L_i \in [0, \underline{R}/(R_e - \underline{R}) \cdot K_i]$. If $L_i > 0$, then $\overline{h} = \underline{h} = R_e$.*
(iii) *If $r < R_e$, then $L_i = lK_i$, where $\{l, \overline{h}, \underline{h}\}$ are determined by the simultaneous equations of (3) and*

$$\underline{h} = \frac{(1 + l)\underline{R}}{l} \tag{6}$$

$$\overline{R} - \overline{h} - \frac{(1 - q)U'(\underline{h})}{qU'(\overline{h})}\frac{\underline{R}}{l} = 0. \tag{7}$$

Moreover, the security to the households is risky: $\underline{h} < r < \overline{h}$.

Proof. See Appendix. □

Results (i) and (ii) are intuitive. The profit margin to the bank of taking in households' funds is $R_e - h_e$. The households, being risk averse, demand a risk premium of $h_e - r \ge 0$. Therefore, if $r > R_e$, banks cannot profit from households' funds. Hence arises result (i). And if $r = R_e$, the profit margin of leverage is at most 0, which occurs only when $h_e = r$, namely with the risk free security: $\overline{h} = \underline{h} = R_e$, which, by (5), implies $L_i \le \underline{R}/(R_e - \underline{R}) \cdot K_i$. This is result (ii).

If $r < R_e$, so long as the bank can offer the risk free security ($\overline{h} = \underline{h} = r$), the profit margin is $R_e - r > 0$. With this security, the bank wants to get as much of households' funds as possible, until (5) is binding, which gives rise to (6) (with $l \equiv L_i/K_i$). Moreover, at the optimum, the security is risky. Start with the risk free contract and now introduce a little risk to it: $\underline{h} = r - \epsilon$ and $\overline{h} = r + (1 - q)/q\epsilon + o(\epsilon)$, which allows for L_i to increase by δL_i, in an order of ϵ, but still keeps the IR, namely constraint (3), satisfied. This variation benefits the bank: the extra cost is of the second order ($qo(\epsilon)$), whereas the gain is $\delta L \cdot (R_e - r - qo(\epsilon))$, of the first order.[3] Lastly, the optimal leverage ratio, L_i/K_i, is independent of K_i because for the bank's contracting problem both the objective Π and the constraints depend on L_i only through the ratio of L_i/K_i.

As (5) is binding, the bank uses all the revenue in the bad state to pay the households, namely, *the optimal contract to households is debt*. And the debt is risky.

Below I discuss properties of the optimal leverage ratio given in result (iii), which is a function of $\overline{R}$, $\underline{R}$ and r and thus denoted by $l(\overline{R}, \underline{R}, r)$. Intuitively, the ratio is driven by a trade-off between the

[2] Intuitively, the bank will not put any funds in storage because the profit margin of doing so is $1 - h_e < 0$.

[3] I thank the referee for the intuition.