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Savings for retirement under liquidity constraints: A note

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ABSTRACT

Pension systems often entail some compulsory saving over which individuals have some degree of choice in terms of the pension plan in which to invest. We analyse whether the choice between alternative plans is affected by the presence of liquidity constraints during working life and we prove that the analytical conditions that determine the choice between different plans are the same in the constrained and unconstrained case.

1. Introduction

Modern pension systems usually entail compulsory savings over which workers have some degree of choice in terms of the pension plan in which to invest. Given the existence of compulsory savings, agents could find it optimal to indebt themselves in order to off-set too large compulsory rates of contribution. Their saving decisions might thus be affected by the presence of incomplete financial markets that prevent them from borrowing the desired amount. Consequentially, liquidity constraints could affect their investment choice. The aim of our work is to analyse what happens to agents' decisions on pension plans when liquidity constraints are binding.

A vast literature has stressed that liquidity constraints affect the amounts saved for retirement (for a review see Magnussen (1994)) but not much has been said on how they affect the destination of those savings. Contributions from Dutta et al. (2000), Wagener (2003), Matsen and Thogersen (2004), De Menil et al. (2006), Corsini and Spataro (2011) and Corsini et al. (2012) cover the topic of decisions on pension plans but none of them focus on the role of liquidity constraints nor examine in details the emergence of corner solutions.

Our contribution extends a model of optimal choice on pension plans developed by Corsini and Spataro (2011) and, differently from it, focuses on the corner solutions that emerge in the presence of liquidity constraints. Our results show that liquidity constraints do not affect the decisions in terms of the pension plan chosen. The implications of our results are twofold. First, from a methodological point of view, the drop of liquidity constraints from this analysis allows for better analytical tractability without any loss in the generality of results. The second implication concerns economic policy: according to our findings, authorities can choose their preferred compulsory contribution rate without worrying that, inducing liquidity constraints, they might somehow bias, through this channel, the choice of individuals' investment plans. In addition, authorities may desire to provide incentives to certain pension plans (for example, to those with higher shares of stocks because this contributes to the development of stock markets or to a higher degree of diversification of individuals' saving portfolios): our result implies that the lessening or tightening of financial constraints (for example, granting easier access to credit services) might not be an effective instrument to achieve this objective.

The work is organized as follows: Section 2 develops the basic model with complete financial markets; Section 3 introduces

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incomplete financial markets and explores the role of liquidity constraints and Section 4 concludes.

2. Saving decisions under complete financial markets

We assume that agents live two periods: in the first they work receiving a wage w and consuming part of their income; in the second they retire, consuming what they have saved. Saving is partly voluntary, cumulated at the risk-free rate r_S, and partly compulsory since a pension system forces individuals to save a fixed contribution rate γ in a pension plan of their choice. For sake of simplicity we imagine that only two plans exist: (i) a safe plan S with the risk-free rate of return r_S and (ii) a risky plan R whose returns are normally distributed with mean r_R and variance σ_R^2. Agents choose how much to consume and to save and which pension plan to adopt. Basically, individuals compute their expected indirect utility under the two plans and then choose the one bestowing the highest indirect utility. Thus we first compute the expected indirect utility for each plan: this is done by solving a maximization problem with respect to first period consumption. With complete financial markets, an individual under the generic plan i faces the following problem:

$$\begin{cases} \max_{c_1} E\left[U\left(c_1, c_2\right)\right] \\ s.t.\ c_2 \sim N\left[\left(w - c_1\right)\left(1 + r_S\right) + \gamma w\left(r_i - r_S\right), \gamma^2 w^2 \sigma_i^2\right] \end{cases} \tag{1}$$

where $E\left[U\left(c_1, c_2\right)\right]$ is the expected lifetime utility that depends on consumption in the two periods (c_1 and c_2 respectively). The constraint in Eq. (1) represents the budget constraint: second period consumption is given only by the returns from compulsory and voluntary saving.

A closed form solution can be obtained assuming the following utility function

$$U = -e^{-ac_1} - \rho e^{-ac_2} \tag{2}$$

where ρ is the rate of time preference and a is the Arrow–Pratt measure of absolute risk aversion.

Following Makarov and Schornick (2010) we assume that a depends on wage with $a = k \cdot w^{-\alpha}$, where $\alpha > 0$ represents the elasticity of risk-aversion-to-wage and k is a positive scale factor. This assumption allows us to obtain decreasing-in-income absolute risk aversion, a property that is usually considered the most realistic one.

Given Eq. (2), we can solve[1] problem (1) for each plan and obtain the following solution in terms of optimal consumption $c_{1,i}^*$ and indirect expected utility $E\left(U_i^*\right)$:

$$c_{1,i}^* = \left[\left(1 + d_i\right) wx - \frac{\log \rho x}{a}\right] \frac{1}{(1+x)} \tag{3}$$

$$E\left(U_i^*\right) = -\left(1 + x\right) x e^{-k(1+d_i)w^{1-\alpha} \frac{x}{1+x}} \rho^{\frac{1}{1+x}} \tag{4}$$

where $x = 1 + r_S$, $d_i = \gamma \frac{r_i - r_S - \gamma k w^{1-\alpha} \sigma_i^2/2}{x}$ and clearly $d_S = 0$.

Individuals choose plan R if $E\left(U_R^*\right) > E\left(U_S^*\right)$ and according to Eq. (4) this inequality is verified if and only if

$$d_R = \gamma \frac{r_R - r_S - \gamma \cdot k \cdot w^{1-\alpha} \cdot \sigma_R^2/2}{x} > 0 \tag{5}$$

thus the sign of d_R determines agents' decisions in terms of pension plans. However, this result is obtained in the absence of liquidity constraints: in the next section we explore the case of liquidity constraints.

[1] Consider that, for any variable z_j distributed normally with mean z and variance σ_z^2 we have $E(e^{-az_j}) = e^{-a(z - a\sigma_z^2/2)}$. See Varian (1992) for details.

3. The role of liquidity constraints

If individuals cannot borrow during their working period, the problem (1) can be restated as

$$\begin{cases} \max_{c_1} E\left[U\left(c_1, c_2\right)\right] \\ s.t.\ c_2 \sim N\left[\left(w - c_1\right)\left(1 + r_S\right) + \gamma w\left(r_i - r_S\right), \gamma^2 w^2 \sigma_i^2\right] \\ s.t.\ c_1 \le \left(1 - \gamma\right) \cdot w \end{cases} \tag{6}$$

where the second constraint represents the non-borrowing condition and implies that first period consumption cannot exceed disposable income and that Eqs. (3) and (4) represent now the inner solution of the problem. In particular (6) has an inner solution for

$$\gamma \le \left(1 + \frac{\log \rho x}{kw^{1-\alpha}}\right) \frac{1}{1+x} - d_i \frac{x}{1+x}. \tag{7}$$

When the above condition does not hold, constraints become binding. Condition (7) depends on the plan chosen so that constraints might be binding under a plan but not under the other. For plan S, we have $d_s = 0$ and Eq. (7) becomes:

$$\gamma \le \left(1 + \frac{\log \rho x}{kw^{1-\alpha}}\right) \frac{1}{1+x}. \tag{8}$$

For plan R we can insert (5) in (7) and obtain the following:

$$kw^{1-\alpha} \sigma^2 \gamma^2/2 - \left(2 + r_R\right)\gamma + \left(1 + \frac{\log \rho x}{kw^{1-\alpha}}\right) \ge 0. \tag{9}$$

The above is a second order equation in γ whose roots are $\gamma_{1,2} = \frac{2 + r_R \pm \sqrt{(2+r_R)^2 - 4\sigma^2\left(kw^{1-\alpha} + \log \rho x\right)}}{kw^{1-\alpha}\sigma^2}$.

Exploiting conditions (8) and (9) we draw in Fig. 1 the couples (w, γ) for which optimal consumption in period one is exactly equal to income. We depict three possible cases depending on the value of the parameter α. Curve S represents the safe plan and is obtained from condition (8) so that, above curve S, liquidity constraints are binding under the plan S. Curves R and R' represent the risky plan and are obtained from condition (9) so that within these two curves liquidity constraints are binding under plan R. Curve D represents the couples (w, γ) for which $d_R = 0$: above it we have $d_R < 0$ while below it we have $d_R > 0$.

Curves S, R and R' define four regions: in region I constraints are not binding for either plans; in region II constraints are binding under both plans; in region III constraints are binding only under plan S and, in region IV, constraints are binding only under plan R. Note that region III completely lies above Curve D and thus within this region we have $d_R < 0$, while for region IV the reverse is true.

Whenever the system is outside region I, condition (7) does not hold for at least a plan and, therefore, optimal consumption and indirect utilities are no longer described by Eqs. (3) and (4) but instead the following corner solutions[2] emerge (the C index denotes the solution when constraints are binding):

$$c_{1,i}^C = \left(1 - \gamma\right) w \quad \forall i \tag{10}$$

$$E\left(U_S^C\right) = -e^{-k(1-\gamma)w^{1-\alpha}} - \rho e^{-k\gamma x w^{1-\alpha}} \tag{11}$$

$$E\left(U_R^C\right) = -e^{-k(1-\gamma)w^{1-\alpha}} - \rho e^{-kw^{1-\alpha} x (d_R + \gamma)}. \tag{12}$$

The above corner solutions show that, in line with previous literature, the amount saved is affected by liquidity constraints.

[2] To obtain (12) consider that

$E(U_R^C) = -e^{-k(1-\gamma)w^{1-\alpha}} - \rho^{\frac{1}{1+x}} e^{-kw^{-\alpha}\left[\gamma w(1+r_R) - kw^{-\alpha}(w\gamma\sigma_R)^2/2\right]}$ and

$\gamma w\left(1 + r_R\right) - kw^{-\alpha}\left(w\gamma\sigma_R\right)^2/2 = wx\left(d_R + \gamma\right).$