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Optimal capital structure with an equity-for-guarantee swap

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ABSTRACT

This paper finds that a newly created equity-for-guarantee swap can significantly increase a firm's value. If the firm earns more/less in a recession/boom market, the guarantee cost will decrease. The greater the business risk is, the more the guarantee cost will decrease and the higher the leverage ratio will be.

1. Introduction

The increasing research interest in small- and medium-sized enterprizes (SMEs) is driven by the recognition that SMEs play a vital role in promoting economic growth. However, there is a scarcity of literature in relation to quantitative research examining the financing problems experienced by SMEs, the financing theory of which is very different from that of a large company. For example, unlike large companies, an SME can seldom issue bonds.

This problem becomes especially serious in an emerging market such as China. To resolve the financing constraints placed on SMEs, certain companies in China, guarantors or insurers, have developed a new financial product, called the *equity-for-guarantee swap* in English. This is an agreement between a bank/lender, an insurer and an SME/borrower, where a bank lends at a given interest rate to an SME and if the SME defaults on the loan, the insurer must pay all the outstanding interest and principal to the bank instead of the SME. In return for the guarantee, the SME must allocate a percentage of the SME's equity to the insurer. Such an innovative contract was first signed in 2002 in the city of Shenzhen in China; recently it has become increasingly popular because it can help SMEs acquire finance easily and cheaply.

While qualitative discussions on the *equity-for-guarantee swap* are available, to our knowledge, this paper is the first to report on a quantitative study of this new financial product and the capital structure of an SME which enters into the swap using a framework developed by Leland (1994).

2. The model

We assume that earnings before interest and tax (EBIT), denoted by η, of an SME/firm is invariant to changes in capital structure and follows an arithmetic Brownian motion given by

$$d\eta = \mu_\eta dt + \sigma_\eta dZ, \quad \eta_0 \text{ given}, \tag{1}$$

where the mean appreciation rate μ_η and volatility $\sigma_\eta > 0$ are constant, and Z is a standard Brownian motion defined on a filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P}\right)$. A filtration $(\mathcal{F}_t)_{t\geq 0}$ describes the flow of information available to an investor.

We take it that the original owners of the SME have chosen a capital structure consisting of equity and of debt in the form of a single console bond, promising a constant coupon payment (C) to the lender, as long as the firm remains solvent. However, unlike the common assumption in the literature, based on many real world situations, we assume that an SME can not issue any bond directly, in contrast to a large company, because of the higher default risk.

To solve the debt financing problem, an SME turns to an insurer and signs a three party agreement with a bank and an insurer. In the agreement, a bank lends at a given interest rate to an SME and

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receives a given constant coupon payment (C). Should the SME default on the loan, the insurer takes possession of the defaulted loan and pays all the outstanding interest and principal to the bank instead of the SME. In return for the guarantee, the SME must allocate a percentage (β) of the SME's equity, called *guarantee cost*, to the insurer. This agreement is actually a swap in finance and so we call it an *equity-for-guarantee swap*.[1]

In addition, similar to Goldstein et al. (2001), we assume a simple tax structure that includes personal and corporate taxes, where interest payments are taxed at a personal rate, τ_i, effective dividends are taxed at τ_d, and corporate profits are taxed at τ_c, with full loss offset provisions.

An extraordinary merit of an equity-for-guarantee swap is that, with the help of the swap, an SME can choose a capital structure consisting of pure equity and of debt in the form of bonds like a large company. On this point, similar to classical corporate finance theory, in this paper we discuss the pricing of corporate securities and optimal capital structure, along with the guarantee cost.

3. Pricing of corporate securities and guarantee cost

Pricing of corporate securities. In finance, most asset prices are derived from a linear pricing schedule.[2] To determine a linear pricing rule, we must specify a *martingale pricing operator*, see Ingersoll (2002), or a *state-price deflator*, e.g., Duffie (2001). However, the market we consider here is incomplete,[3] i.e. there are infinite state-price deflators. To fix one, we can solve a single-agent optimization problem and take the marginal utility of the agent as the special state-price deflator, see Duffie (2001). If the agent selected is the representative agent, then we specify the equilibrium martingale pricing operator, as used in Ingersoll (2002), Goetzmann et al. (2003), and essentially also in Merton (1976). With regard to the problem we discuss here, if ρ denotes the correlation coefficient between the EBIT and the market portfolio, $\eta^M \equiv (\mu_M - r)/\sigma_M$ the Sharpe ratio of the market, and $\mu \equiv \mu_\eta - \rho\sigma_\eta\eta^M$ the risk-adjusted mean appreciation rate of the EBIT of the firm, then under this particularly chosen state-price deflator, thanks to (1), the EBIT η of an SME follows a risk-adjusted arithmetic Brownian motion given by

$$d\eta = \mu dt + \sigma_\eta dZ^{\mathbb{Q}}, \quad \eta_0 \text{ given}, \tag{2}$$

where $Z^{\mathbb{Q}}$ is a Brownian motion under the risk-neutral measure $\mathbb{Q}$ that corresponds to the chosen deflator.[4] Therefore, at any time $t \geq 0$ according to (2) and we can compute the value V_t of the EBIT in the following way

$$V(\eta_t) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^\infty e^{-r(s-t)}\eta_s ds|\mathcal{F}_t\right] = \frac{\mu}{r^2} + \frac{\eta_t}{r}, \tag{3}$$

where r is the risk-free rate. Under the specific state-price deflator chosen above, the value $F(\eta)$ of any time-independent claim underlying the EBIT with a constant payment flow CF to the claimant must satisfy

$$\mu F_\eta + \frac{\sigma_\eta^2}{2}F_{\eta\eta} + CF = rF. \tag{4}$$

[1] The new financial product, equity-for-guarantee swap, we discuss here shares many similarities with a credit default swap (CDS), but there are also significant differences. For example, the former gives partial equity to an insurer instead of a series of payments (the CDS "fee" or "spread") made to a CDS seller, see Rutkowski (2009) among others.

[2] For example, utility indifference prices are nonlinear.

[3] This is because the risk of the EBIT cannot be completely hedged by a portfolio in general, i.e. the EBIT cannot be replicated.

[4] For the relationship between state-price deflators and risk-neutral measures, please refer to the Proposition in Section 6F in Duffie (2001).

The general solution of the homogenous equation of (4) is given by

$$F(\eta) = A_1\exp(-k_1\eta) + A_2\exp(-k_2\eta),$$
$$k_{1/2} = \frac{\mu \mp \sqrt{\mu^2 + 2r\sigma_\eta^2}}{\sigma_\eta^2}, \tag{5}$$

where A_1 and A_2 are constants, determined by some boundary conditions.

Following that, for all $t \geq 0$ and $\eta_B \in (0, \eta_t)$, where η_B is considered a default threshold that triggers the bankruptcy of the SME, fix the value $p_B(\eta_t, \eta_B)$ of a security that claims one unit of account at the hitting time $\tau_B \equiv \inf\{u \geq t;\ \eta_u \leq \eta_B\}$. According to (5), the value of this claim is given by

$$F(\eta_t, \eta_B) \equiv p_B(\eta_t, \eta_B) = \exp(-k_2(\eta_t - \eta_B)). \tag{6}$$

Moreover, the market value of equity, denoted by $E(\eta_t, \eta_B, C)$, satisfies the following differential equation,

$$\mu E_\eta + \frac{\sigma_\eta^2}{2}E_{\eta\eta} + (1-\tau_f)(\eta - C) = rE, \tag{7}$$

with the following value-matching and smooth-pasting conditions,

$$E(\eta_B, \eta_B, C) = 0, \qquad E'(\eta_B, \eta_B, C) = 0, \tag{8}$$

where the effective tax rate τ_f is given by $(1-\tau_f) = (1-\tau_c)(1-\tau_d)$. Solving Eq. (7) with boundary conditions (8), we obtain

$$E(\eta_t, \eta_B, C) = (1-\tau_f)\left[\frac{\mu + r\eta_t}{r^2} - \frac{\mu + r\eta_B}{r^2}p_B(\eta_t, \eta_B) - \frac{C}{r}(1 - p_B(\eta_t, \eta_B))\right]. \tag{9}$$

For a given coupon payment C, assume that equity holders can choose the bankruptcy threshold η_B, then the optimal bankruptcy threshold η_B^* is obtained by the following condition

$$\left.\frac{\partial E(\eta_t, \eta_B, C)}{\partial \eta_B}\right|_{\eta=\eta_B^*} = 0. \tag{10}$$

Thus, we easily obtain the following optimal bankruptcy threshold

$$\eta_B^* = C - \mu/r - 1/k_2. \tag{11}$$

In the same way, the market value of debt without insurance, denoted by $D(\eta_t, \eta_B, C)$, is given by

$$D(\eta_t, \eta_B, C) = (1-\tau_i)\left[\frac{C}{r}(1 - p_B(\eta_t, \eta_B)) + (1-\alpha)\frac{\mu + r\eta_B}{r^2}p_B(\eta_t, \eta_B)\right], \tag{12}$$

where α is the bankruptcy loss rate (i.e. distress rate). The bankruptcy loss rate α can be interpreted in different ways, such as loss from selling real assets, asset fire-sale losses, legal fees, etc.

Pricing of guarantee cost. Although the volume of equity-for-guarantee swaps traded is increasing rapidly in China, the question of how much guarantee cost should be charged to receive a loan has never been studied formally. Now we are ready to ask such a question.

We denote by D_{guar} the value of the insurer's compensatory payment to the bank/lender, which is taxed at a personal rate τ_i. For a given coupon C, the bankruptcy threshold η_B^* is given by (11). Hence, in order to fully protect the debt, the value D_{guar} must satisfy the following equation

$$D(\eta_t, \eta_B^*, C) + (1-\tau_i)D_{guar} = D_0(C) \equiv \frac{C}{r}(1-\tau_i), \tag{13}$$