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Disagreement, correlation and asset prices

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ABSTRACT

When people agree to disagree, how does the disagreement affect asset prices? Within an equilibrium framework with two agents, two risky assets and a riskless bond, we analyze the joint impact of disagreement about expected payoff, variance and correlation, and compare prices with benchmark prices in a market with homogeneous beliefs.

1. Introduction

In financial markets, it is well recognized that people agree to disagree and the disagreement can have a significant impact on asset prices (see for example Fama and French, 2007). Disagreement complicates the formulation of asset prices, which makes a complete analysis difficult. In a static setting, when investors with the same risk tolerance agree on the covariance matrix, several authors have shown that assets remain correctly priced and the disagreement effect "cancels out" when beliefs about expected returns are heterogeneous but on average unbiased (see for example Levy et al., 2006 and Yan, 2010). The analysis becomes much more complicated when there is a disagreement about the covariance matrix, because investors' demands are non-linear functions of their beliefs of the covariance matrix. Recently, Chiarella et al. (2011) showed that, when asset payoffs are uncorrelated, disagreement about variances leads to a diversification effect. However, Duchin and Levy (2010) show that tiny fluctuations in the disagreement about the variance lead to substantial price fluctuations. Moreover, most of the literature focuses on the price impact of a specific type of disagreement (expected returns or variances) by assuming investors are otherwise identical, and not much attention has been paid to their joint impact, which can be very different from their individual impact. For example, Jouini and Napp (2006, 2008) and Chiarella et al. (2011) find that the impact of disagreement on prices is governed by the risk tolerance weighted average level of pessimism/optimism.

In a market with two risky assets, agents may have different *risk tolerances*, and jointly disagree about the *expected payoffs*, *variances of payoffs* and the *correlation between payoffs*. We show that even when agents have the same objective belief about the expected payoff and variance for the *first* asset, the market as a whole can be overoptimistic/overpessimistic and overconfident about its payoff if agents simultaneously disagree about the expected payoff and variance of the *second* asset *or* simultaneously disagree about the expected payoff of the second asset and the correlation between payoffs. As a result, prices of both assets are in general different from the benchmark prices in a market with homogeneous beliefs. This leads to a *spillover effect* of disagreement in a multi-asset market. All our results are limited to a static model. Impact of disagreement in a dynamic model can be very different. For example, Jouini and Napp (2011) show that even when beliefs are on average unbiased and risk tolerances are the same, disagreement can have a significant impact on the price dynamics and the risk–return trade-off of risky assets.

This paper is organized as follows, Section 2 presents an equilibrium asset pricing model with heterogeneous beliefs, Section 3 analyzes the impact of disagreement on asset prices and Section 4 concludes.

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2. The model

We consider a two-date economy with two risky assets, indexed by $k = 1, 2$, a riskless bond, and two agents, indexed by $i = 1, 2$. The bond is in zero net supply and each agent is endowed with one share of each risky asset on date zero. The *future payoff* of asset k is denoted by X_k and let $\mathbf{X} = (X_1, X_2)^T$, moreover, the *risk-free rate* is assumed to be zero and the current price of the bond is 1. The asset payoffs are assumed to be *jointly normal* and agents are assumed to have *heterogeneous beliefs* about the *expected payoffs* and *covariance matrix of the payoffs*. For agent i, $(i = 1, 2)$, let

$$\boldsymbol{\mu}_i \equiv (\mu_{i,1}, \mu_{i,2})^T, \qquad \mathbf{V}_i \equiv \begin{pmatrix} \sigma_{i,1}^2 & \rho_i\sigma_{i,1}\sigma_{i,2} \\ \rho_i\sigma_{i,1}\sigma_{i,2} & \sigma_{i,2}^2 \end{pmatrix},$$

where $\mu_{i,k} = \mathbb{E}_i(X_k)$, $\sigma_{i,k}^2 = \mathrm{Var}_i(X_k)$, $\rho_i = Correl_i(X_1, X_2)$ for $i, k = 1, 2$, and denote $\mathcal{B}_i := (\boldsymbol{\mu}_i, \mathbf{V}_i)$ the subjective belief of agent i.

2.1. Portfolio optimization

The terminal wealth of agent i is given by $W_i = z_{i,B} + \mathbf{z}_i^T\mathbf{X}$, where $\mathbf{z}_i = (z_{i,1}, z_{i,2})^T$ is the number of shares of the risky assets held by agent i, and $z_{i,B}$ is the number of bonds held. Agent i maximizes a *constant absolute risk aversion* (CARA) utility function $U_i(W_i) = -\tau_i \exp\{-W_i/\tau_i\}$ of his terminal wealth W_i under his subjective belief $\mathcal{B}_i$, subject to the budget constraint $\mathbf{z}_i^T\mathbf{p} + z_{i,B} = \mathbf{p}^T\mathbf{1}$, where τ_i is agent i's risk-tolerance. When the terminal wealth W_i is normally distributed, maximizing $\mathbb{E}_i[U_i(W_i)]$ is equivalent to maximizing the *certainty equivalent wealth* given by $z_{i,B} + \mathbf{z}_i^T\boldsymbol{\mu}_i - \frac{1}{2\tau_i}\mathbf{z}_i^T V_i \mathbf{z}_i$, where $\mathbf{p} = (p_1, p_2)^T$ is the equilibrium price vector of the risky assets. Therefore, the optimal portfolio of agent i is given by

$$\mathbf{z}_i^* = \tau_i \mathbf{V}_i^{-1}(\boldsymbol{\mu}_i - \mathbf{p}) \quad \text{and} \quad z_{i,B}^* = \mathbf{p}^T(\mathbf{1} - \mathbf{z}_i^*). \tag{1}$$

2.2. Consensus belief and market equilibrium

The market clearing conditions are given by $\frac{1}{2}(\mathbf{z}_1^* + \mathbf{z}_2^*) = \mathbf{1}$ and $z_{1,B} + z_{2,B} = 0$. Note that agents' budget constraints imply that

$$\mathbf{p}^T\mathbf{1} = \frac{1}{2}(\mathbf{z}_1^* + \mathbf{z}_2^*)^T\mathbf{p} + \frac{1}{2}(z_{1,B} + z_{2,B}). \tag{2}$$

Therefore, the bond market clears as long as the asset market clears.

To characterize market equilibrium under heterogeneous beliefs, a concept of consensus belief has been developed by Lintner (1969) and Rubinstein (1974, 1975). In this paper, a belief $\mathcal{B}_a = (\boldsymbol{\mu}_a, \mathbf{V}_a)$ is called a market *consensus belief* if the equilibrium prices under the heterogeneous beliefs $\mathcal{B}_i := (\boldsymbol{\mu}_i, \mathbf{V}_i)(i = 1, 2)$ are also the equilibrium prices under the homogeneous belief $\mathcal{B}_a$.

We construct a consensus belief similar to Chiarella et al. (2011), which allows us to analyze the heterogeneous economy as an equivalent homogeneous economy. Let $\tau_a = \frac{1}{2}(\tau_1 + \tau_2)$ be the *average* risk tolerance. Applying Proposition 3.2 in Chiarella et al. (2011), the consensus belief $\mathcal{B}_a$ is given by

$$\begin{aligned} \mathbf{V}_a^{-1} &= \frac{1}{2}\left[\frac{\tau_1}{\tau_a}\mathbf{V}_1^{-1} + \frac{\tau_2}{\tau_a}\mathbf{V}_2^{-1}\right], \\ \boldsymbol{\mu}_a &= \frac{1}{2}\left[\frac{\tau_1}{\tau_a}(\mathbf{V}_a\mathbf{V}_1^{-1})\boldsymbol{\mu}_1 + \frac{\tau_2}{\tau_a}(\mathbf{V}_a\mathbf{V}_2^{-1})\boldsymbol{\mu}_2\right]; \end{aligned} \tag{3}$$

and the equilibrium asset prices are given by

$$\mathbf{p} = \boldsymbol{\mu}_a - \mathbf{V}_a\mathbf{1}/\tau_a. \tag{4}$$

Furthermore, the equilibrium optimal portfolio of agent i is given by

$$\mathbf{z}_i^* = \tau_i V_i^{-1}[(\boldsymbol{\mu}_i - \boldsymbol{\mu}_a) + V_a\mathbf{1}/\tau_a]. \tag{5}$$

In the following, we use the consensus belief constructed in Eq. (3) to examine the impact of disagreement among agents on the equilibrium prices (4) of risky assets.

3. The price impact of disagreements

To measure the price impact of disagreement, we first consider a benchmark economy in which agents have homogeneous beliefs and the same level of risk tolerance, that is, $\mathcal{B}_i = \mathcal{B}_o = (\boldsymbol{\mu}_o, \mathbf{V}_o)$, where $\mathcal{B}_o$ may be regarded as the *objective belief* about the distribution of asset payoffs and $\tau_i = \tau$. Since there is no disagreement, the consensus belief in this case coincides with the objective belief, that is

$$\boldsymbol{\mu}_a = \boldsymbol{\mu}_o \equiv (\mu_1, \mu_2)^T, \qquad \mathbf{V}_a = \mathbf{V}_o \equiv \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

and the equilibrium asset prices under homogeneous belief, or the *benchmark prices*, denoted by $\hat{\mathbf{p}}$ are given by

$$\hat{\mathbf{p}} = (\mu_1 - (\sigma_1^2 + \rho\sigma_1\sigma_2)/\tau,\ \mu_2 - (\sigma_2^2 + \rho\sigma_1\sigma_2)/\tau)^T. \tag{6}$$

For the economy with heterogeneous beliefs, we assume that agents agree about the expected payoff and standard deviation of the first asset (S_1), that is, $(\sigma_{i,1}, \mu_{i,1}) = (\sigma_1, \mu_1)$ for $i = 1, 2$. Furthermore, there is a disagreement about the expected payoff and standard deviation of the second asset (S_2), and also the correlation between asset payoffs. The disagreement among agents is measured by

$$\Delta\mu \equiv \mu_{1,2} - \mu_{2,2}, \qquad \Delta\sigma \equiv \sigma_{1,2} - \sigma_{2,2}, \quad \text{and}$$
$$\Delta\rho \equiv \rho_1 - \rho_2.$$

When $\Delta\mu > (<)0$, agent 1 is relatively more optimistic (pessimistic) about the payoff of S_2 than agent 2; when $\Delta\sigma > (<)0$, agent 1 is relatively more doubtful (confident) about the payoff of S_2 than agent 2; when $\Delta\rho > (<)0$, agent 1 perceives a higher (lower) correlation between asset payoffs than agent 2. Moreover, assume the average risk tolerance is given by $\tau_a = \tau$, the difference in risk tolerance is measured by $\Delta\tau \equiv \tau_1 - \tau_2$. Hence, when $\Delta\tau > (<)0$, agent 1 is more (less) risk tolerant than agent 2. Following (4), the equilibrium prices are then determined by the consensus belief,

$$\begin{aligned} \mathbf{p} = (&\mu_{a,1} - (\sigma_{a,1}^2 + \rho_a\sigma_{a,1}\sigma_{a,2})/\tau, \\ &\mu_{a,2} - (\sigma_{a,2}^2 + \rho_a\sigma_{a,1}\sigma_{a,2})/\tau)^T. \end{aligned}$$

If consensus belief coincides with the objective belief, then $\mathbf{p} = \hat{\mathbf{p}}$.

To facilitate the analysis, we introduce notations of three different averages, namely the *arithmetic, geometric* and *harmonic* averages, defined by

$$A(x_1, x_2) \equiv (x_1 + x_2)/2, \qquad G(x_1, x_2) \equiv \sqrt{x_1 x_2},$$
$$H(x_1, x_2) \equiv [(1/x_1 + 1/x_2)/2]^{-1}.$$

Note that, when $x_1 \neq x_2$, we have $H(x_1, x_2) < G(x_1, x_2) < A(x_1, x_2)$. To examine the impact of the disagreement, we consider three cases.

Case 1. The impact of risk tolerance and optimism/pessimism— This case has been considered in the literature. For example, in a market with a single risky asset, Jouini and Napp (2007) show that the consensus belief of the expected payoff is a risk-tolerance weighted average of agents' perceived expected payoffs. We show in the next proposition[1] that this result also carries over to a multi-asset market.

Proposition 1. *When $\Delta\sigma = \Delta\rho = 0$, the consensus belief is given by $\mathbf{V}_a = \mathbf{V}_o$, $\boldsymbol{\mu}_a = (\mu_1, \alpha\mu_{1,2} + (1 - \alpha)\mu_{2,2})^T$, where*

[1] Proofs of propositions only involve simple algebra, therefore are omitted from the paper.