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Participation in deterministic contests

Ron Siegel

Department of Economics, Northwestern University, United States

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ABSTRACT

This paper considers participation in deterministic contests for $m \geq 1$ identical prizes, which employ an auction-like rule to determine the winners. In most papers that investigate such models, participation by precisely $m + 1$ players is associated with players having complete information about their opponents' characteristics, and participation by more than $m+1$ players is associated with players having incomplete information about their opponents' characteristics. I show that incomplete information is in fact neither sufficient nor necessary for participation by more than $m + 1$ players.

1. Introduction

Some real-world competitions induce active participation by many competitors, whereas others lead to active participation by only a few competitors, often only one more than the number of prizes. What determines the number of active participants in a competition for $m \geq 1$ identical prizes? More than $m + 1$ competitors may participate if, given their investments, there is exogenous uncertainty regarding the outcome. This is the case, for example, in Tullock's (1980) lottery model, in which each player's probability of winning is proportional to his share of the aggregate investments. When the outcome given investments is deterministic, as is the case in auction-like contest models, participation by more than $m + 1$ competitors occurs when competitors have private information about their valuations. This is the case, for example, in the classic symmetric independent private value all-pay auction (Krishna, 2002). In contrast, precisely the $m+1$ competitors with the highest valuations participate in the complete-information all-pay auction when players' valuations differ (Clark and Riis, 1998).[1] Participation by only the $m + 1$ strongest players is also a hallmark of complete-information variants of the all-pay auction (examples include Che and Gale (1998) and González-Díaz (forthcoming)). Thus, it is natural to ask whether equilibrium participation by more than $m + 1$ competitors in deterministic models of competition occurs, at least "generically", if and only if competitors are uncertain about some characteristics of their rivals.

This paper includes two results, Propositions 1 and 2, which together show that the answer is "no". The first result shows that for any number of players, prizes, and participants (larger than the number of prizes), there exist complete-information, deterministic contests with these number of players and prizes in which in any equilibrium precisely the specified number of players participate. This generalizes Siegel's (2009) example in which three players participate in a complete-information contest for one prize. The result is robust in that it continues to hold when the contests are perturbed slightly. The second result identifies a large class of incomplete-information deterministic contests in which only the strongest $m + 1$ players participate. This class includes multiprize all-pay auctions in which players' valuations are drawn from disjoint intervals. Intuitively, these results show that participation by many players does not stem from incomplete information per se, but arises when different players are known to have sufficient cost advantages in different regions of the competition.

2. Complete-information contests

In a *contest*, n players compete for m homogeneous prizes, $0 < m < n$. The set of players $\{1, \ldots, n\}$ is denoted by $\mathcal{N}$. Every player i chooses a score s_i from $\mathbb{R}_+ = [0, \infty)$. Given $\mathbf{s} = (s_1, \ldots, s_n)$, where s_i is player i's chosen score, player i's *payoff* is

$$u_i(\mathbf{s}) = P_i(\mathbf{s}) V_i - c_i(s_i),$$

where $V_i > 0$ is player i's *valuation for a prize*, $c_i \colon \mathbb{R}_+ \to \mathbb{R}$ is player i's continuous, strictly increasing *cost function*, with $c_i(0) = 0$ and

E-mail address: r-siegel@northwestern.edu.

[1] Baye et al. (1996) showed that more than two competitors may participate when certain players' valuations are identical. This participation result is not robust to slight changes in players' valuations.

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$\lim_{s_i \to \infty} c_i(s_i) > V_i$, and $P_i: \mathbb{R}^n_+ \to [0, 1]$ is player i's *probability of winning*, which satisfies

$$P_i(\mathbf{s}) = \begin{cases} 0 & \text{if } s_j > s_i \text{ for } m \text{ or more players } j \neq i, \\ 1 & \text{if } s_j < s_i \text{ for } n - m \text{ or more players } j \neq i, \end{cases}$$

and $\sum_{j=1}^{n} P_j(\mathbf{s}) = m$. When all costs are linear, we have an all-pay auction (Hillman and Samet, 1987; Hillman and Riley, 1989; Clark and Riis, 1998).[2] The primitives of the contest are commonly known.

Definition 1. (i) Player i's *reach* r_i is the score whose cost equals the player's valuation for a prize. That is, $r_i = c_i^{-1}(V_i)$. Re-index players in (any) decreasing order of their reach, so that $r_1 \geq r_2 \geq \cdots \geq r_n$.
(ii) Player $m + 1$ is the *marginal player*.
(iii) The *threshold* T of the contest is the reach of the marginal player: $T = r_{m+1}$.

A final requirement in the definition of a contest is that only the marginal player's reach equals the threshold. In an all-pay auction, for example, a player's reach is his valuation for a prize, the marginal player is the player with the $(m + 1)$st highest valuation, and the marginal player's valuation is required to be different from those of the other players. The model of contests described here is a special case of Siegel's (2009) all-pay contest model.

A player *participates* in an equilibrium of a contest if with positive probability he chooses positive scores (whose cost is positive).[3]

Proposition 1. *For any n, m, and k such that $n \geq k > m > 0$, there exist contests with n players and m prizes such that in any equilibrium precisely k players participate.*

The proof of Proposition 1 is in the Appendix. Intuitively, Proposition 1 stems from the fact that many players participate when different players have local cost advantages in different regions. To see why this happens, consider a player i whose cost in a certain interval I of scores is much lower than those of the other players. Lemma 1 in the Appendix shows that at least two players choose scores in I. If player i does not participate, then his payoff is 0. But because other players choose scores in I, by doing so they have to win with a probability that is sufficiently high to offset their costs. And because player i's costs are much lower then those of the other players, player i would obtain a positive payoff by choosing these scores in I. Therefore, player i must participate.

Baye et al. (1996) showed that many players may participate in certain complete-information, single-prize all-pay auctions in which many players have the same valuation for the prize.[4] Their finding differs from Proposition 1 in two ways. First, the auction games of Baye et al. (1996) have many equilibria, and in contrast to Proposition 1, the number of players that participate differs across equilibria, and equals two in some equilibria. Second, perturbing players' valuations in an all-pay auction leads to a unique equilibrium, in which only the two players with the highest valuations participate. The contests constructed in the proof of Proposition 1 are robust to such perturbations: small changes in players' cost functions or valuations do not change players' participation.

[2] In an all-pay auction, $c_i(s_i) = s_i$ and ties are resolved by randomizing uniformly.

[3] A player wins a prize with positive probability if and only if he participates. Indeed, because participation is costly and choosing 0 is not, a participating player must win a prize with positive probability. In the other direction, the Tie Lemma in Siegel (2009) and the fact that 0 is the lowest possible score imply that a player who chooses 0 wins a prize with probability 0.

[4] These all-pay auctions do not meet the definition of a contest because players other than the marginal player have reaches that equal the threshold.

3. Incomplete-information contests

Take a contest (as defined in Section 2) in which V_i and c_i are commonly known for every player i. Players are indexed as in Definition 1. Now, add incomplete-information in the following way. Every player i's valuation is $V_i + \varepsilon_i$, where ε_i is player i's private information and is drawn from $[-\delta, \delta]$ for some $\delta > 0$ according to some distribution μ_i. Players' cost functions remain commonly known. Each player, after observing his private information, chooses a score, and the winners are determined as in the complete-information case.

Proposition 2. *If for some $i > m + 1$ we have*

$$\frac{c_{m+1}(x)}{V_{m+1}} < \frac{c_i(x)}{V_i} \quad \text{for all } x > 0, \tag{1}$$

then for small $\delta > 0$ player i does not participate in any equilibrium of the incomplete-information contest described above. In particular, if this condition holds for every player $i = m + 2, \ldots, n$, then in any equilibrium only players $1, \ldots, m + 1$ may participate.

The proof of Proposition 2 is in the Appendix. The logic underlying Proposition 2 is as follows. The marginal player can be shown to have a payoff of 0 in any equilibrium when he has his lowest possible valuation. Suppose that some player $i > m+1$ that satisfies the conditions of Proposition 1 participates, and consider a positive score chosen by player i in equilibrium. By choosing this score when he has his highest possible valuation, player i obtains a non-negative payoff, and therefore wins with a probability that is sufficiently high to offset his costs. Because the marginal player's costs are strictly lower than those of player i, the marginal player when he has his lowest possible type can obtain a positive payoff by choosing a score slightly higher than the highest score chosen by player i, a contradiction.

As an application of Proposition 2, take a complete-information all-pay auction in which the marginal player's valuation differs from those of all other players, and add some incomplete information as specified above. Proposition 2 shows that, for $\delta > 0$ that is not too large, players $m + 2, \ldots, n$ do not participate in any equilibrium of the incomplete-information contest. The proof of Proposition 2 shows that this is true for any $\delta < \min\left\{\frac{V_m - V_{m+1}}{2}, \frac{V_{m+1} - V_{m+2}}{2}\right\}$.

Appendix. Proofs of Propositions 1 and 2

A.1. Notation and existing results

The following three results refer to complete-information contests, and are immediate corollaries of results in Siegel (2009).[5] I use these results in the proofs of Propositions 1 and 2 below. The first result characterizes players' equilibrium payoffs in terms of their *power*, where player i's power w_i is his payoff if he chooses the threshold and wins: $w_i = V_i - c_i(T)$.

Theorem 1. *In any equilibrium of a contest, the expected payoff of every player equals the maximum of his power and* 0.

In addition to giving a closed-form formula for players' equilibrium payoffs, Theorem 1 shows that players $1, \ldots, m$ have positive expected payoffs, and players $m + 1, \ldots, n$ have expected payoffs of 0.

[5] The results follow, respectively, from Theorems 1 and 2, and Lemma 1 in Siegel (2009).