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The incentive effect of a handicap

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A B S T R A C T

The incentive effect of a handicap in a tournament competition is studied. A handicap may decrease the effort levels of the advantaged group or the disadvantaged group. However, the average effort level will always increase as long as the performance measure is informative of effort in a specific sense: the monotone likelihood ratio property.

1. Introduction

A handicap is an advantage given to a weak player to level the field in a game or sport. This practice is quite prevalent in physical sports such as golf, polo, and horse racing, as well as in brain sports such as chess and Go. A similar idea has been extended to a social context, and part of the rationale behind an affirmative action policy is to compensate for the historical disadvantage of minority groups and level the field.

The purpose of handicaps seems to be two-fold. First, a handicap tries to reduce the gap in winning probability among competitors and to make the result less predictable. Second, it induces competitors to exert more effort.

The effect of handicap has usually been studied in a one-on-one tournament competition,[1] and the existing literature seems to verify these two purposes of a handicap. Shotter and Weigelt (1992) shows theoretically and experimentally that a handicap will increase the effort levels of both competitors by reducing the gap in winning probability.[2] Fu (2006), using the all pay auction framework, also shows that the effort levels of both competitors increase as the gap in winning probability is reduced and are maximized when the winning probability is equalized.

However, not all handicaps and affirmative action policies occur in the context of one-on-one competition and the implications of the literature may be limited in different contexts. For example, affirmative action in university admissions is a handicap applied to a tournament of many competitors.

This paper studies the incentive effect of handicaps in a tournament of many competitors, which seems to be relevant to affirmative action in a social context. Two groups of competitors participate in the competition: advantaged and disadvantaged. We first show that the effort incentive can be captured by a simple term: probability of being on the margin. When a competitor's performance is on the margin of winning, a small change in effort can tip the performance for winning. Second, the incentive effect of the handicap for each group cannot be definitely predicted. A handicap can decrease the effort incentives of one group, and its incentive effect depends on the situation. Third, the general level of effort will always increase with the handicap as long as performance is informative of effort. In other words, we provide a very general condition for the positive incentive effect of handicaps.

This paper is organized as follows. Section 2 introduces the model setting. The equilibria before and after the implementation of the handicap are analyzed in Section 3. The effort incentives are compared in Section 4. The conclusion follows.

2. Model

There is a unit mass of competitors. Each competitor is denoted by $i \in [0, 1]$. These competitors are divided into two groups: $j = 0, 1$. The mass of Group 0 is α and that of Group 1 is $1-\alpha$. Group

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[1] For the discussion of tournament competition, see Lazear and Rosen (1981) and O'Keeffe et al. (1984).

[2] Shotter and Weigelt's original theoretical prediction was that effort levels will be reduced by a handicap. However, this prediction was wrong and later corrected by Fain (2009). This theoretical result is dependent on the error term distribution.

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1 is advantaged in the sense that they have better performance on average with the same effort level, which will be specified later.

Winners of the competition are chosen by agents' performances p_i, which are determined by his/her pre-given (or pre-acquired) ability a_i and effort level e_i with some errors. Specifically, the performance is the sum of a_i, e_i, and the error term ε_i[3]:

$$p_i = a_i + e_i + \varepsilon_i.$$

Agents are identical in their pre-given abilities in the same group. Let $a_i = a_0$ if i is in group 0 and $a_i = a_1$ if i is in group 1. Group 1 is more advantaged since this pre-given ability is higher, $a_1 > a_0$. For simplicity, we normalize $a_0 = 0$.

Agents can change their performances by exerting efforts, but with cost. The cost of effort is given by $c(e)$, with $c' > 0$ and $c'' > 0$.

The error term ε_i is assumed to be iid over $(-\infty, \infty)$ with a probability density function $f(\varepsilon)$ and a distribution function $F(\varepsilon)$.

The competition will select W mass of agents as winners and give the prize of utility value V. We normalize the utility value of losing the competition to be 0. As winners will be selected by performance, the top W mass in the performance distribution will be chosen to be winners.

A handicap in this competition means giving more favor to the disadvantaged group 0, i.e., we add some points to the performance level of group 0 agents. When we mention handicap h, it means that h is added to the performances of Group 0 agents.

3. Equilibrium

3.1. When there is no handicap

In the competition, agents will choose their effort levels to maximize the expected utility. We restrict our attention to a symmetric equilibrium, where identical agents in the same group choose the same level of effort. Suppose that Group 0 agents exert effort e_0 and Group 1 agents e_1.

The cutoff γ will emerge in the performance distribution and the mass of agents, whose performances are higher than γ, is W. Since we have a continuum of agents, the ex-post performance distribution is the same as the ex-ante distribution. Therefore, the cutoff γ is determined by

$$\alpha\left[1 - F(\gamma - e_0)\right] + (1-\alpha)\left[1 - F(\gamma - a_1 - e_1)\right] = W. \tag{1}$$

Effort choices e_0 and e_1 should be optimal with the given cutoff γ. The expected utility of each agent is

$$U_i = V \cdot \Pr\{p_i \geq \gamma\} - c(e_i)$$
$$= \begin{cases} V\left[1 - F(\gamma - e_i)\right] - c(e_i) \\ \quad \text{if the agent is in group 0} \\ V\left[1 - F(\gamma - a_1 - e_i)\right] - c(e_i) \\ \quad \text{if the agent is in group 1,} \end{cases}$$

as the probability of being a winner is

$$\Pr\{p_i \geq \gamma\} = \Pr\{\varepsilon_i \geq \gamma - a_i - e_i\}$$
$$= 1 - F(\gamma - a_i - e_i).$$

Thus, the following first-order condition should be satisfied at the supposed effort levels e_0 and e_1;

$$Vf(\gamma - e_0) = c'(e_0) \tag{2}$$
$$Vf(\gamma - a_1 - e_1) = c'(e_1).$$

The left-hand side of Eq. (2) is the expected marginal benefit. If the agent marginally increases his/her effort level, whether s/he will be the winner is affected only when s/he is on the cutoff. If the realization of the error term is too favorable, the agent will be the winner even without the marginal effort. In contrast, if the realization of the error term is too unfavorable, the agent will lose the competition, even with the marginal effort. When s/he is on the cutoff, the marginal effort will change the outcome and make the agent a winner with utility value V. Therefore, the expected marginal benefit is the value of prize V times the probability of being on the cutoff $f(\cdot)$. The first order condition states that the marginal benefit should be equal to the marginal cost of effort at the optimum.

The (symmetric) equilibrium of this competition is characterized by triples (γ, e_0, e_1), which satisfy Eqs. (1) and (2). To avoid the need to check the second-order condition, we assume that c'' is large enough. In this setting, a unique equilibrium exists.

Proposition 1 (*Existence*)**.** *A unique equilibrium exists.*

Proof. In the Appendix. □

Hereafter, we abuse the notation and also denote the equilibrium by (γ, e_0, e_1) if it does not cause confusion. We also define $\varepsilon_0 \equiv \gamma - e_0$ and $\varepsilon_1 \equiv \gamma - a_1 - e_1$; minimum realizations of the error term to be a winner for each group.

A simple revealed preference argument can show that exerted efforts cannot overtake the difference in pre-given abilities. If a performance level is worthwhile for Group 0 agents to achieve, then it is also worthwhile for Group 1 agents to achieve as they can achieve it with even less effort. Therefore, the order of the average performance is always preserved.

Lemma 1. *In equilibrium, the average performance of Group 1 agents is higher than that of Group 0 agents, or $a_1 + e_1 > e_0$.*

Proof. In the Appendix. □

A Group 1 agents' choice problem is the same as a Group 0 agents' except that the relevant cutoff is $\gamma - a_1$ instead of γ. The above lemma also implies that the change of effort cannot offset the change of cutoff. Therefore, as the cutoff decreases, the probability of winning always increases.

Corollary 1. *As the cutoff γ increases (decreases), the chance of being selected decreases (increases) as $\varepsilon \equiv \gamma - e$ increases (decreases). That is, the change of the effort level cannot offset the change of the cutoff.*

Note that this result does not say anything about the change in the effort level except that it cannot offset the change of the cutoff. However, this result will be useful when comparing the equilibria before and after handicapping is implemented, which is our next discussion.

3.2. When handicap h is implemented

We now add h to the performances of the Group 0 agents as a handicap. We assume that this handicap does not eliminate the difference in the pre-given abilities ($h < a_1$).

We denote a new equilibrium $\left(\gamma^h, e_0^h, e_1^h\right)$ where superscript h denotes the handicap. Let $\varepsilon_0^h \equiv \gamma^h - h - e_0^h$ and $\varepsilon_1^h \equiv \gamma^h - a_1 - e_1^h$. Then equilibrium satisfies

$$\alpha\left[1 - F\left(\varepsilon_0^h\right)\right] + (1-\alpha)\left[1 - F\left(\varepsilon_1^h\right)\right] = W \tag{3}$$
$$Vf\left(\varepsilon_0^h\right) = c'\left(e_0^h\right)$$
$$Vf\left(\varepsilon_1^h\right) = c'\left(e_1^h\right).$$

Hereafter, we will construct the relationship between equilibria before and after the handicap.

[3] Though we assume the effort affects the achievement in a linear fashion, assuming general function $g(e)$ in the achievement level would not affect the logic of the main analysis.