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Decomposing abnormal returns in stochastic linear models☆

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ABSTRACT

This paper presents a method helpful in analyzing the sources of return in an event study. A generalized decomposition result derived from the differential between two random linear functions attributes the effect of events or regulations on the value of firms to differences in economy-wide and individualistic factors. In aggregate decomposition, the abnormal return in the existing literature is equivalent to the coefficient effects. As an example, I take the market model in Card and Krueger (1995) showing that this approach helps provide additional insights.

1. Introduction

Social scientists are often interested in the economic effects of events and government regulations. Does a regulation confer net benefits on consumers at the expense of regulated firms? Do regulated firms receive net benefits at the expense of consumers? Our interest is in making predictions about the effects of event or regulation on the value of the regulated firms.

The effects of the event/regulation on firms are typically obtained by estimating abnormal returns arising from the event/regulation. The abnormal returns are then aggregated to draw overall inferences for the effects of the event/regulation. We can further ask where the effects come from and how can their sources be identified? It is of interest to learn how much of the effects originates from the issuer of the security (individualistic factors) and how much is attributable to factors that affect securities in general (economy-wide factors).

This paper presents a method of analyzing the sources of return in an event study. A generalized decomposition result derived from the differential between two random linear functions attributes the effect of events or regulations on the value of the firms to differences in economy-wide and individualistic factors. In aggregate decomposition, I show that the abnormal return in the existing literature is equivalent to the coefficient effects. Section 2 presents the theoretical framework. In Section 3, I take the market model in Card and Krueger (1995) showing that this approach helps provide additional insights. Section 4 is the concluding remarks.

2. Theoretical framework

2.1. Decomposing returns

Decomposition techniques for linear regression models have been widely used in social research for many decades. The technique utilizes the output from regression models to parcel out components of a group difference in a statistic (such as a mean or proportion) which can be attributed to differences between groups and to differences in the effects of characteristics (Powers et al., 2011). In this section, I introduce a method allowing decomposition of the firm's returns into differences in economy-wide and individualistic factors.

To illustrate the idea, Fig. 1 shows the time line for the estimation and event windows used to decompose the returns of a firm in an event study. Following the notations in Campbell et al.

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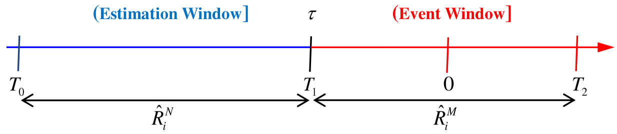

Fig. 1. Time line for decomposing returns in an event study.

(1997), let τ be the time index so I define $\tau = T_0 + 1$ to $\tau = T_1$ as the estimation window used to estimate the normal return, $\hat{R}_i^N$, of firm i. Define $\tau = T_1 + 1$ to $\tau = T_2$ as the event window and $\tau = 0$ as the event day. Let $L_1 = T_1 - T_0$ and $L_2 = T_2 - T_1$ be the length of the estimation and the event window, respectively. $\hat{R}_i^M$ is the estimated event-window return of firm i from $T_1 + 1$ to T_2 during the event window. Estimated return after T_2, the post-event window, can also be obtained if applicable.

Suppose the return-generating process of firm i on day τ is

$$R_{i\tau} = X_{i\tau}\beta_i + \varepsilon_{i\tau}. \tag{1}$$

The equation identifies the return $R_{i\tau}$ as a linear combination of $X_{i\tau}$ and β_i, where $R_{i\tau}$ is the return of firm i on day τ, $X_{i\tau} = \begin{bmatrix} 1 & X_{i\tau}^1 \cdots X_{i\tau}^K \end{bmatrix}$ is a $(1 \times (K+1))$ row vector of factors with one in the first column, $\beta_i = \begin{bmatrix} \alpha_i & \beta_i^1 \cdots \beta_i^K \end{bmatrix}'$ is a $((K+1) \times 1)$ column vector of parameters with an intercept α_i and coefficients β_i^1 to β_i^K. $\varepsilon_{i\tau}$ is the disturbance term. Express Eq. (1) as a regression system,

$$\mathbf{R}_i = \mathbf{X}_i\boldsymbol{\beta}_i + \boldsymbol{\varepsilon}_i,$$

and let N denote normal returns and M denote estimated event-window returns. In the estimation and event windows, the return-generating processes can be written as

$$\begin{aligned} \mathbf{R}_i^N &= \mathbf{X}_i^N\boldsymbol{\beta}_i^N + \boldsymbol{\varepsilon}_i^N \\ \mathbf{R}_i^M &= \mathbf{X}_i^M\boldsymbol{\beta}_i^M + \boldsymbol{\varepsilon}_i^M, \end{aligned} \tag{2}$$

where $\mathbf{R}_i^N = \begin{bmatrix} R_{iT_0+1}^N \cdots R_{iT_1}^N \end{bmatrix}'$ is an $(L_1 \times 1)$ normal returns vector in the estimation window and $\mathbf{R}_i^M = \begin{bmatrix} R_{iT_1+1}^M \cdots R_{iT_2}^M \end{bmatrix}'$ is an $(L_2 \times 1)$ estimated event-window returns vector in the event window. $\mathbf{X}_i^N = \begin{bmatrix} \iota & X_{i\tau}^{jN} \end{bmatrix}$ is an $(L_1 \times (K+1))$ matrix with a vector of ones ι in the first column and a matrix of factors $X_{i\tau}^{jN}$ with $j = 1, \ldots, K$ and $\tau = T_0 + 1, \ldots, T_1$ in the estimation window. $\mathbf{X}_i^M = \begin{bmatrix} \iota & X_{i\tau}^{jM} \end{bmatrix}$ is an $(L_2 \times (K+1))$ matrix with a vector of ones ι in the first column and a matrix of factors $X_{i\tau}^{jM}$ with $j = 1, \ldots, K$ and $\tau = T_1 + 1, \ldots, T_2$ in the event window. $\boldsymbol{\beta}_i^N = \begin{bmatrix} \alpha_i^N & \beta_i^{1N} \ldots \beta_i^{KN} \end{bmatrix}'$ and $\boldsymbol{\beta}_i^M = \begin{bmatrix} \alpha_i^M & \beta_i^{1M} \ldots \beta_i^{KM} \end{bmatrix}'$ are the $((K+1) \times 1)$ parameter vectors for the estimation and event windows, respectively. Finally, $\varepsilon_i^N = \begin{bmatrix} \varepsilon_{iT_0+1}^N \cdots \varepsilon_{iT_1}^N \end{bmatrix}'$ is the $(L_1 \times 1)$ vector of disturbance terms of the normal return equation and $\varepsilon_i^M = \begin{bmatrix} \varepsilon_{iT_0+1}^M \cdots \varepsilon_{iT_1}^M \end{bmatrix}'$ is the $(L_2 \times 1)$ vector of disturbance terms of the estimated event-window return equation.

Define DR_i as the difference in returns of firm i which is the mean outcome difference between the estimated event-window return and the normal return,

$$DR_i = E(\mathbf{R}_i^M) - E(\mathbf{R}_i^N), \tag{3}$$

where $E(\cdot)$ denotes the expected value of the outcome variable. Under the least squares assumptions, $E(\boldsymbol{\beta}_i) = \boldsymbol{\beta}_i$ and $E(\boldsymbol{\varepsilon}_i) = \mathbf{0}$. So equations in (2) can be shown as

$$\begin{aligned} E(\mathbf{R}_i^N) &= E(\mathbf{X}_i^N)\boldsymbol{\beta}_i^N \\ E(\mathbf{R}_i^M) &= E(\mathbf{X}_i^M)\boldsymbol{\beta}_i^M. \end{aligned} \tag{4}$$

Using the sample means $\bar{\mathbf{X}}_i^N$ and $\bar{\mathbf{X}}_i^M$ as estimates for $E(\mathbf{X}_i^N)$ and $E(\mathbf{X}_i^M)$ and the least squares estimates $\hat{\boldsymbol{\beta}}_i^N$ and $\hat{\boldsymbol{\beta}}_i^M$ for $\boldsymbol{\beta}_i^N$ and $\boldsymbol{\beta}_i^M$, DR_i in Eq. (3) can be written as

$$\begin{aligned} DR_i &= E(\mathbf{R}_i^M) - E(\mathbf{R}_i^N) \\ &= \bar{\mathbf{X}}_i^M\hat{\boldsymbol{\beta}}_i^M - \bar{\mathbf{X}}_i^N\hat{\boldsymbol{\beta}}_i^N \\ &= (\bar{\mathbf{X}}_i^M\hat{\boldsymbol{\beta}}_i^M - \bar{\mathbf{X}}_i^M\hat{\boldsymbol{\beta}}_i^N) + (\bar{\mathbf{X}}_i^M\hat{\boldsymbol{\beta}}_i^N - \bar{\mathbf{X}}_i^N\hat{\boldsymbol{\beta}}_i^N) \\ &= \underbrace{(\bar{\mathbf{X}}_i^M - \bar{\mathbf{X}}_i^N)\hat{\boldsymbol{\beta}}_i^N}_{\text{Endowment/Characteristics Effects}} + \underbrace{\bar{\mathbf{X}}_i^M(\hat{\boldsymbol{\beta}}_i^M - \hat{\boldsymbol{\beta}}_i^N)}_{\text{Coefficient Effects}}. \end{aligned} \tag{5}$$

Eq. (5) is the generic result of decomposing returns. The first component in (5) contains the endowment/characteristics effects which explain how much of the difference in returns is attributable to differences in endowment/characteristics. The second component contains the coefficient effects which explain how much of the difference in returns is attributable to differences in coefficients.

The next step is to find the contribution of each factor to DR_i. Applying the detailed decomposition method proposed in Yun (2004, 2005), Eq. (5) can be written as

$$DR_i = \sum_{j=1}^{K} W_{\Delta X}^j \left[(\bar{\mathbf{X}}_i^M - \bar{\mathbf{X}}_i^N)\hat{\boldsymbol{\beta}}_i^N\right] + \sum_{j=1}^{K} W_{\Delta\beta}^j \left[\bar{\mathbf{X}}_i^M(\hat{\boldsymbol{\beta}}_i^M - \hat{\boldsymbol{\beta}}_i^N)\right],$$

where

$$\sum_{j=1}^{K} W_{\Delta X}^j = \frac{(\bar{X}_i^{jM} - \bar{X}_i^{jN})\hat{\beta}_i^{jN}}{(\bar{\mathbf{X}}_i^M - \bar{\mathbf{X}}_i^N)\hat{\boldsymbol{\beta}}_i^N},$$

$$\sum_{j=1}^{K} W_{\Delta\beta}^j = \frac{\bar{X}_i^{jM}(\hat{\beta}_i^{jM} - \hat{\beta}_i^{jN})}{\bar{\mathbf{X}}_i^M(\hat{\boldsymbol{\beta}}_i^M - \hat{\boldsymbol{\beta}}_i^N)}, \quad \text{and}$$

$$\sum_{j=1}^{K} W_{\Delta X}^j = \sum_{j=1}^{K} W_{\Delta\beta}^j = 1.$$

In the existing literature of an event study, the analysis computes potential abnormal returns (which play a crucial role) using residuals in the event/post-event window. Let AR_i be the abnormal returns of firm i which is the residuals computed with coefficients estimated using data from the estimation window as

$$AR_i = \mathbf{R}_i^* - (\bar{\mathbf{X}}_i^M\hat{\boldsymbol{\beta}}_i^N),$$

where $\mathbf{R}_i^*$ is the observed returns vector of firm i. Under the case where $\mathbf{R}_i^* = \hat{\mathbf{R}}_i^M$, Eq. (5) becomes

$$\begin{aligned} DR_i &= (\bar{\mathbf{X}}_i^M - \bar{\mathbf{X}}_i^N)\hat{\boldsymbol{\beta}}_i^N + (\hat{\mathbf{R}}_i^M - \bar{\mathbf{X}}_i^M\hat{\boldsymbol{\beta}}_i^N) \\ &= (\bar{\mathbf{X}}_i^M - \bar{\mathbf{X}}_i^N)\hat{\boldsymbol{\beta}}_i^N + AR_i, \end{aligned} \tag{6}$$

which shows that the abnormal return is equivalent to the coefficient effects in the aggregate decomposition.

2.2. Significance tests for difference in returns

Since $\bar{\mathbf{X}}_i$ and $\hat{\boldsymbol{\beta}}_i$ are uncorrelated by assumption and assuming that $\mathbf{R}_i^N$ and $\mathbf{R}_i^M$ are independent, the mean and the variance[1] for DR_i are

$$\mathbf{E}(DR_i) = (\bar{\mathbf{X}}_i^M - \bar{\mathbf{X}}_i^N)\hat{\boldsymbol{\beta}}_i^N + \bar{\mathbf{X}}_i^M(\hat{\boldsymbol{\beta}}_i^M - \hat{\boldsymbol{\beta}}_i^N),$$

[1] Brown and Rutemiller (1977) present a method of estimating the mean and variance of a linear function with arbitrary multivariate randomness in its coefficients and variables.