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Forecasting the yield curve for the Euro region

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ABSTRACT

This paper compares the forecast precision of the Functional Signal plus Noise (FSN), the Dynamic Nelson–Siegel (DL), and a random walk model. The empirical results suggest that both outperform the random walk at short horizons (one-month) and that the FSN model outperforms the DL at the one- and three-months forecasting horizon. The conclusions provided in this paper are important for policy makers, fixed income portfolio managers, financial institutions and academics.

1. Introduction

The accurate forecast of the yield curve has gained significant importance in the last few years. The yield curve has proved to be a leading indicator for economic activity and inflation. It also has a massive influence over the development of macroeconomic scenarios, which are employed by large companies, financial institutions, regulators and institutional investors.

The main purpose of this paper is to evaluate the forecast performance of the two leading models presented in the literature for the yield curve denominated in Euros, namely the dynamic version of the parametrically parsimonious Nelson and Siegel (1987) model presented by Diebold and Li (2006) (DL) and the Functional Signal plus Noise with an Error Correction Model (FSN-ECM) introduced by Bowsher and Meeks (2008). Therefore, we will compare both these models, as well as report their comparative results against a random walk (RW) forecast for one-, three- and six-months forecasting horizon. It is worth mentioning that while the DL model has been already tested, the only information (due to the authors) that we have about the FSN-ECM model is that it is competitive for short-term horizon forecasting. In particular, Vicente and Tabak (2008) have shown that the DL model is very superior for forecasting purposes to the models with affine term structure.

We contribute to the literature in two main ways. First we compare different methods to forecast the Euro yield curve and find evidence that the FSN-ECM performs better in the short run than the DL model. Second, we employ a recent data-set that includes the last few years, in which yields have declined substantially due to the crisis that hit the US and global markets in 2007 and 2008. Therefore, the forecasting properties of these models are tested within a period in which substantial changes in the yield curve have taken place.

The remainder of the paper is structured as follows. Section 2 provides a description of the methods used to construct yield curve forecasts. Section 3 describes the data-set employed in the analysis. The empirical results are presented in Section 4. Section 5 concludes the paper.

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2. Methodology

In this section, we present the forecast models used in this paper and also the Diebold–Mariano statistic, which is used to compare predictive accuracy.

2.1. Forecast models

In this paper, we consider three forecast models, namely random walk, the Diebold–Li (Diebold and Li, 2006) and the Bowsher–Meeks models (Bowsher and Meeks, 2008).

2.1.1. Random-walk model

A random walk (RW) without drift for the yield of each individual maturity τ is given by

$$y_t(\tau) = y_{t-1}(\tau) + \varepsilon_t(\tau), \tag{1}$$

where $\varepsilon_t(\tau) \sim N(0, \sigma^2(\tau))$ is a white noise process.

Therefore, the h-period-ahead yield forecast is given by

$$\hat{y}_{t+h}(\tau) = y_t(\tau), \tag{2}$$

i.e., it is equal to the most recent observed value of $y_t(\tau)$.

2.1.2. Diebold–Li model

The Diebold and Li (2006) method follows the Nelson and Siegel (1987) exponential components framework to distill the entire yield curve, period by period, into a three-dimensional parameter that evolves dynamically. The corresponding yield curve is

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \tag{3}$$

where t is the date and τ is the maturity.

The parameter λ governs the exponential decay rate: small values of λ produce slow decay and can better fit the curve at long maturities, while large values of λ produce fast decay and can better fit the curve at short maturities. We follow Diebold and Li (2006) and adopt λ as a constant given by the value that maximizes the loading on the medium-term factor, as shown in Eq. (3).

The terms β_{1t}, β_{2t} and β_{3t} are interpreted as three latent dynamic factors. The loading on β_{1t} is a constant, 1, that does not decay to 0 at the limit; hence, it is viewed as a long-term factor. The loading on β_{2t} is a function that starts at 1 and decays monotonically and quickly to 0; hence it may be viewed as a short-term factor. Finally, the loading on β_{3t} starts at 0, increases, and then decays to zero; hence it may be viewed as a medium-term factor. These three factors may also be interpreted in terms of level, slope and curvature respectively.

Because we have fixed the value of λ, we are able to compute the values of the two regressors and estimate the factors (β) for each period t by Ordinary Least Squares. By doing so, we create a time series estimates of $\left\{\hat{\beta_{1t}}, \hat{\beta_{2t}}, \hat{\beta_{3t}}\right\}$. We now forecast the values of the factors using a univariate AR(1). The yield forecasts $\hat{y}_{t+h/t}(\tau)$ based on the AR(1) factor specifications are given by Eq. (3) with β_{it} replaced by $\hat{\beta}_{i,t+h/t}$, for $i = 1, 2, 3$, where

$$\hat{\beta}_{1,t+h/t} = \hat{c}_\imath + \hat{\gamma}_\imath \hat{\beta}_{\imath t}, \quad \imath = 1, 2, 3$$

and h is the forecast horizon.

2.1.3. Bowsher–Meeks model

The FSN-ECM model consists of a dynamically evolving natural cubic spline signal function denoted by $S_{\gamma_t}(\boldsymbol{\tau})$, plus a noise process, whose dynamic evolution is driven by a cointegrated VAR in the form of error correction model. A cubic spline is essentially a piecewise cubic function with pieces that join to form a smooth function. The spline signal function, $S_{\gamma_t}(\boldsymbol{\tau}) := (S_{\gamma_t}(\tau_1), \ldots, S_{\gamma_t}(\tau_N))'$, has m knots, positioned at the maturities $k = (1, k_2, \ldots, k_m)$, which are deterministic and fixed over time. The notation $S_{\gamma_t}(\boldsymbol{\tau})$ is used to imply that the spline interpolates to the latent yields $\gamma_t = (\gamma_{1t}, \ldots, \gamma_{mt})'$—i.e. $S_{\gamma_t}(k_\jmath) = \gamma_{\jmath t}$ for $\jmath = 1, \ldots, m$. We refer to the vector γ_t as the *knot yields* of the spline.

The model for the time series of N-dimensional observed yield curves, $\{y_t(\boldsymbol{\tau})\}$, is given by

$$\begin{aligned} y_t(\boldsymbol{\tau}) &= S_{\gamma_t}(\boldsymbol{\tau}) + \epsilon_t \\ &= W(k; \boldsymbol{\tau})\gamma_t + \epsilon_t, \end{aligned}$$

$$\Delta\gamma_{t+1} = \alpha(\beta'_{\gamma_t} - \mu_s) + \Psi\Delta\gamma_t + \nu_t.$$

Here $S_{\gamma_t}(\boldsymbol{\tau})$ is a natural cubic spline on $(k; \gamma_t)$, the $N \times m$ deterministic matrix $W(k; \boldsymbol{\tau})$ is defined as $S_{\gamma_t}(\boldsymbol{\tau})/\gamma_t$, the $m \times (m-1)$ matrix α has full rank, and the matrix β is defined uniquely by $\beta'_{\gamma_t} = (\gamma_{\jmath+1,t} - \gamma_{\jmath t})_{\jmath=1}^{m-1}$. The initial state $(\gamma'_1, \gamma'_0)'$ has finite first and second moments given by μ^* and Ω^* respectively. The Gaussian FSN has the additional condition that $(\gamma'_1, \gamma'_0)'$ have multivariate Normal distributions.

The parameters of the various FSN models are estimated by maximizing the likelihood of the corresponding Gaussian FSN model, computed using the Kalman filter. The FSN forecasts are the 1-step-ahead point predictions given by the Kalman filter, $[\hat{y}_t(\boldsymbol{\tau})|y_{t-1}(\boldsymbol{\tau}), \ldots, y_1(\boldsymbol{\tau}); \theta]_{KF}$, with the parameter vector of the model set equal to some estimated value, θ. In this case, $[\hat{y}_t(\boldsymbol{\tau})|y_{t-1}(\boldsymbol{\tau}), \ldots, y_1(\boldsymbol{\tau}); \theta]_{KF}$ is a linear function of the past observations $(y_{t-1}(\boldsymbol{\tau}), \ldots, y_1(\boldsymbol{\tau}))$ and has minimum MSFE amongst the class of such linear predictors when θ is equal to the true parameter vector.

The forecasts are defined by a vector $\varphi_t := Q\gamma_t$ consisting of the (latent) short rate and inter-knot (latent) yield spreads:

$$\begin{aligned} \varphi_t &:= \left(\gamma_{1t}, \gamma_{2t} - \gamma 1t, \ldots, \gamma_{mt} - \gamma_{m-1,t}\right)' \\ &= \begin{pmatrix} 1 & 0_{1\times(m-1)} \\ \beta' & \end{pmatrix} \gamma_t = Q\gamma_t. \end{aligned}$$

The state equation may then be written equivalently as the VAR

$$\Delta\varphi_{t+1} = Q\alpha(\beta' Q^{-1}\varphi_t - \mu_s) + Q\Psi Q^{-1}\Delta\varphi_t + \eta_t$$

where $\eta_t = Q\nu_t$.

Two important points here are the choice of the number of knots and the position of the knots. Since one may interpret the FSN-ECM as a special type of dynamic factor model in which the knots of the splines are the factors and we are comparing this model with the Diebold–Li three-factor model, we present the results only for the model with three knots. However, we compared these results with the four-knot model and the results are very similar. In order to choose the position of the knots, we used the in-sample cross-sectional regression procedure considered in Bowsher and Meeks (2008).

We also estimate the FSN model without the error correction term. In this case the knot-yields follow an unrestricted VAR. The Diebold–Li model also employs an unrestricted VAR in the latent factors. Therefore, we compare both FSN and FSN-ECM with the Diebold–Li model.

2.2. Diebold–Mariano statistic

Let y_t be the series to be forecast and let $y^1_{t+h/t}$ and $y^2_{t+h/t}$ be two competing forecast models. Using the time series y_t and the forecast models, the forecast errors of the two models are given by

$$\epsilon^i_{t+h/t} = y_{t+h} - y^i_{t+h/t}, \quad \text{for } i = 1, 2.$$