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Multi-attribute procurement auctions with risk averse suppliers

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ABSTRACT

We analyze multi-attribute procurement auctions with risk-averse suppliers. As the number of suppliers increases or the suppliers become more risk-averse, the equilibrium bidding price decreases under the first-score auction but remains the same under the second-score auction. A buyer prefers the first-score auction.

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1. Introduction

Procurement auctions are widely applied in government procurements and private enterprise purchases. Unlike traditional forward auctions, a bid in procurement auctions often involves price and non-price attributes, such as quality, time of delivery and service levels. Therefore, researchers consider procurement auctions as *multi-attribute*. Thus far, extensive research has been done on risk neutral procurement auctions (Branco, 1997; Che, 1993; Samuelson, 1986). But, as pointed out by Maskin and Riley (1984), the marginal utility of income if a bidder wins is often not the same as that if he loses. Indeed, suppliers of procurement contracts are shown to be risk averse (Campo, 2009). Unfortunately, the existing literature on risk averse procurement is quite limited. Holt (1980) considered risk averse suppliers in a single-attribute auction setting. Baron and Besanko (1987) characterized the optimal procurement contract for a monopsonistic buyer who contracts with a single risk averse supplier. There has not been a systematic analysis of the role of supplier risk aversion in multi-attribute procurement auctions.

The goal of this article is to examine a multi-attribute procurement auction model with risk averse suppliers. In particular, we are interested in how risk aversion and number of suppliers affect auction outcomes and which payment rule (first- or second-score) the buyer should use. Our contribution lies in extending the analysis of multi-attribute procurement auctions to the risk averse case. We analytically establish that buyer procurement cost decreases with risk aversion and number of suppliers. Finally, we show that the first-score auction is preferred to the second-score auction with risk averse suppliers.

2. The model

A buyer wishes to acquire a single commodity (or service) from one of n suppliers. The buyer cares about the cost of the acquisition and the *quality* of the acquired commodity, such as build quality, time of delivery, and other non-monetary attributes. For simplicity, we assume that quality is a single-dimensional variable, denoted as q. It will be clear later (see footnote 2) that changing to multi-dimensional quality does not alter the qualitative nature of our findings.

The buyer uses a procurement auction to solicit bids from the n suppliers. Each bid is a *pair* (p, q) that consists of a price p and a quality q. A supplier incurs a cost of $c(q, \theta)$ to produce quality q. As in procurement auction literature (Che, 1993), the cost parameter θ is the supplier's private information and is independently and identically distributed on $[\underline{\theta}, \overline{\theta}]$ according to

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a distribution function $F(\cdot)$ with a positive and continuously differentiable density $f(\cdot)$. We assume that the supplier's cost is increasing in quality q and cost parameter θ and convex in q, i.e., $c_q(\cdot,\cdot) > 0$, $c_\theta(\cdot,\cdot) > 0$, and $c_{qq}(\cdot,\cdot) \geq 0$. An example of such a cost function is $c(q,\theta) = q\theta$.

Let $u(\cdot)$ denote the supplier's von Neumann–Morgenstern utility for income. A supplier's expected utility is

$$U(p,q) = u(\text{payment} - c(q,\theta)) \times \text{Prob}\{\text{win}|p,q\} \tag{1}$$

where $\text{Prob}\{\text{win}|p,q\}$ is the winning probability of bid (p,q). We assume $u(0) = 0$ and $u'(x) > 0$ for all $x \geq 0$. $u(\cdot)$ can be linear or concave, which models *risk neutral* and *risk averse* suppliers respectively.

The buyer is risk neutral and his utility is given by a quasi-linear function

$$M(\text{payment}, q) = m(q) - \text{payment} \tag{2}$$

where $m'(\cdot) > 0$ and $m''(\cdot) < 0$. We assume $m'(0) > c_q(0,\cdot)$ and $m'(\infty) < c_q(\infty,\cdot)$ to ensure that each supplier would choose a finite positive quality upon winning.

The buyer ranks bids using a publicly announced scoring rule that is consistent with his utility.[1] Namely, the *score* of bid (p,q) is given by

$$S(p,q) = M(p,q) = m(q) - p. \tag{3}$$

The supplier with the highest score wins the auction (ties are broken by coin toss). As in the literature (Che, 1993; Branco, 1997), the winner with a bid of (p,q) receives a payment of p and delivers quality q in the first-score auction. The same winner needs to "fulfill" the second highest score in the second-score auction in the sense that the winner is free to choose any quality and payment combination as long as the score of his chosen combination equals the second highest score.

3. Results

3.1. Supplier's quality choice

Lemma 1. *In the first- and second-score auctions, a supplier with cost parameter θ will choose the equilibrium quality $Q(\theta)$ according to*

$$Q(\theta) \equiv \arg\max_q [m(q) - c(q,\theta)]. \tag{4}$$

Proof. See Che (1993) for the proof of a similar result for the first-score auction. For the second-score auction, suppose the contrary that the supplier with cost parameter θ chooses to bid (p,q) in equilibrium, where $q \neq Q(\theta)$. We show that the supplier is strictly better off with an alternative bid $(p^*, Q(\theta))$ where $p^* = p + m(Q(\theta)) - m(q)$. In fact, it follows from (3) that $S(p^*, Q(\theta)) = S(p,q)$ and then $\text{Prob}\{\text{win}|p^*, Q(\theta)\} = \text{Prob}\{\text{win}|p,q\}$. Moreover, we have $m(q) - S_2 - c(q,\theta) < m(Q(\theta)) - S_2 - c(Q(\theta),\theta)$, where S_2 is the second highest score and the inequality is true because $m(q) - c(q,\theta)$ has unique interior maximum at $Q(\theta)$ by assumptions about $m(\cdot)$ and $c(\cdot,\cdot)$. Now the expected utilities for the two bids have the following relationship:

$$\begin{aligned} EU(p^*, Q(\theta)) &= u(m(Q(\theta)) - S_2 - c(Q(\theta),\theta)) \\ &\quad \times \text{Prob}\{\text{win}|p^*, Q(\theta)\} \\ &> u(m(q) - S_2 - c(q,\theta)) \times \text{Prob}\{\text{win}|p,q\} \\ &= EU(p,q) \end{aligned}$$

which suggests that (p,q) cannot be an equilibrium bid. □

Lemma 1 implies that in both first-score and second-score auctions, the supplier's equilibrium quality choice is a function of his cost parameter θ and the scoring rule, but is independent of his equilibrium price. Furthermore, a supplier chooses quality to maximize the total surplus created by (4). Lemma 1 allows us to transform the multi-attribute procurement auction into a single attribute one.

3.2. Supplier's equilibrium price choice

Denote

$$v = V(\theta) \equiv m(Q(\theta)) - c(Q(\theta),\theta) \tag{5}$$

as the "valuation" of a supplier with cost parameter θ. By the envelope theorem, $V'(\theta) = -c_\theta(Q(\theta),\theta) < 0$. So $V(\theta)$ is strictly decreasing with θ. Let $\bar{v} = V(\underline{\theta})$ and $\underline{v} = V(\bar{\theta})$, then v is distributed on $[\underline{v}, \bar{v}]$ according to

$$H(v) \equiv 1 - F(V^{-1}(v)) \tag{6}$$

with a density $h(v) = H'(v)$. We further denote the "bid score" of $(p, Q(\theta))$ as

$$b \equiv m(Q(\theta)) - p. \tag{7}$$

It is straightforward to verify that with transformations (5) and (7), the first- and second-score auctions correspond to standard first- and second-price auctions respectively with valuation v and bid $b(v)$.[2] v corresponds to the surplus created in the original auctions, $b(v)$ corresponds to the equilibrium bidding score, and the equilibrium bid prices can be easily derived from (7) and (4).

Theorem 1. *In the first-score auction, the equilibrium bidding price is given by*

$$P_{FS}(\theta) = m(Q(\theta)) - b(V(\theta)) \tag{8}$$

where the equilibrium score $b(\cdot)$ is determined by the following differential equation:

$$b'(v) = \frac{(n-1)h(v)u(v - b(v))}{H(v)u'(v - b(v))} \tag{9}$$

with a boundary condition $b(\underline{v}) = \underline{v}$.

Proof. Following the existing methods (e.g., Riley and Samuelson, 1981) for solving standard first auctions with risk averse bidders, we can obtain (9) in the transformed first-price auction. (8) follows directly from (7). □

Theorem 2. *In the second-score auction, it is a weakly dominant strategy for the suppliers to quote costs as prices where*

$$P_{SS}(\theta) = c(Q(\theta),\theta). \tag{10}$$

Proof. It directly follows from (7) and the known result that it is a weakly dominant strategy for risk averse bidders to bid truthfully in standard second-price auctions (e.g., McAfee and McMillan, 1987). □

It is clear from Theorem 2 that the risk aversion and number of bidders have no effect on suppliers' bids in the second-score auction. But the same may not be true in the first-score auction.

[1] The buyer may not creditably commit to other scoring rules because it is optimal for him to select the bids based on his true utility after receiving all the bids.

[2] In the case of multi-dimensional quality, say, $q = (q_1, q_2, \ldots, q_k)$, we can similarly obtain an optimal quality vector $Q(\theta) = \arg\max_q [m(q) - c(q,\theta)]$. Once again, we can transform the multi-attribute procurement auction to a standard auction using (5) and (7). Our subsequent results are generalizable to multi-dimensional quality.