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Auctions with both common-value and private-value bidders

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ABSTRACT

This paper shows the existence of monotone pure-strategy equilibrium in auctions with both common-value bidders and private-value ones. In equilibrium, the common-value bidders bid less aggressively when there are more private-value bidders. Further, resale is discussed as an application.

1. Introduction

In many practical auctions, both common-value bidders and private-value bidders participate together. Take gallery auctions in New York as an example: East coast and west coast are the two main markets for art works. While consumers living in the east coast might join the New York auctions directly, it is more costly for consumers living in the west coast to join the auctions. Instead, some investors such as gallery dealers and shop owners from the west coast might bid in New York auctions, and resell the items to west coast consumers. Thus such auctions in New York include two different kinds of bidders: private-value bidders who are mostly consumers living in the east coast and common-value bidders who are investors from the west coast.

Auctions with different types of bidders have been actively studied in the recent years. Maskin and Riley (2000) discussed asymmetric auctions with "strong" and "weak" private-value bidders. Lebrun (1999) solved first-price auctions with asymmetric independent private values. While both of these papers only consider asymmetric bidders with pure private values. Reny and Zamir (2004) proved the existence of equilibrium in general asymmetric first-price auctions, while they also pointed out that their method is not suitable for second-price auctions. Hafalir and Krishna (2008) showed the asymmetry in bidders' private values leads to inefficient allocation and post-auction trade (resale).

Moreover, the literature has also discussed the combination of common values and private values. Goeree and Offerman (2002, 2003) studied the model where the values have both private and common components, and further did an experiment to explore the inefficiency in such auctions. However, the equilibria in such auctions do not always exist, such as Jackson (2009) showed that with discrete private and common values, the equilibrium may not exist. The combination is private and common components in values in all these papers. While in this paper we discuss auctions with both common-value bidders (investors) and private-value bidders (consumers).

The main result of this paper is the existence of monotone pure-strategy equilibrium in such second-price auctions. In equilibrium, common-value bidders bid less aggressively when increasing the number of private-value bidders.

Then as an application we include the resale period, in which an investor can resell the item to some other consumers if she wins in the first-period auction. We consider the efficiency based on the final allocation of the item to the consumers. The probability of having efficiency loss converges to a positive number when increasing the number of consumers, so we cannot perfectly avoid such loss. While the good news is that the expected efficiency loss and expected profit loss shrink to zero.

The structure of this paper is as follows: Section 2 discusses the model of auctions with both kinds of bidders, describes the equilibrium, provides the equilibrium characterization and explicitly discusses an example. Section 3 introduces the resale procedure and analyzes the efficiency. The last section briefly concludes the paper.

2. Model and equilibrium

There are one seller, M common-value bidders and N private-value bidders in a standard second-price auction for a single indivisible

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item. Each private-value bidder j has a value $Z_j \in [\underline{z}, \overline{z}]$, and Z_j are identically independently distributed with the density $f(z)$ and c.d.f. $F(z)$. The common value $q \in [\underline{z}, \overline{z}]$ has the same possible support as the private values[2]. Each common-value bidder i does not know the exact q, but observes a realization x_i of her random signal $X_i \in [\underline{x}, \overline{x}]$. The true value q is strictly positively correlated with $(X_1,...,X_M)$, representing that larger signals imply that the true value is more likely to be larger than smaller. Moreover, conditional on any realization of X_i all other signals are identically independently distributed with the density $g(x|x_i)$ and c.d.f. $G(x|x_i)$[3]. All these densities are in C_1[4]. The common value and the signals $(q,X_1,...,X_M)$ are independent of all private values. All the above are common knowledge except for the private values and private signals. The seller and all bidders are risk neutral.

In this setting, the private-value bidders always bid their true values because it is their dominant strategy in second-price auctions. Now consider a common-value bidder i with a signal x. Taking other common-value bidders' bidding strategy is $\beta(\cdot)$ as given, by bidding ρ, i's expected payoff is:

$$\Pi(x,\rho) = \int_{\underline{b}}^{\rho} (v(x,b,\beta(\cdot)) - b)\, d\xi(b)$$

in which $v(x,b,\beta(\cdot))$ is i's expected common value when winning the auction, given her signal (x), the highest bid among others' (b) and their bidding strategy (β). The F.O.C. is $b(x) = v(x,b(x),\beta(\cdot))$, described by the following lemma:

Lemma 1. If the symmetric monotone equilibrium bidding strategy $b^*(x)$ exists and all other common-value bidders use this strategy, a common-value bidder with signal x will bid b equal to the expected common value $v(x,b,b^*(\cdot))$, given that her signal is x and the highest bid among others' is b.

Now we describe $v(x,b,\beta(\cdot))$, assuming the other common-value bidders use strictly monotone and continuous strategy $\beta(\cdot)$. Denote Y_i to be the highest signal observed by all other common-value bidders except i. Note that there are two possibilities for b, which is the highest bid from all other bidders:

(A) b is from a common-value bidder: The expected common value is $E(q|A) = E(q|X_i = x, Y_i = \beta^{-1}(b))$, denoted as $H(x,\beta^{-1}(b))$. The probability of this is $Pr(A|X_i = x) = g_{(1)}(\beta^{-1}(b)|x) F_{(1)}(b)\frac{d\beta^{-1}(b)}{db}$[5].

(B) b is from a private-value bidder: The expected common value is $E(q|B) = E(q|X_i = x, Y_i \le \beta^{-1}(b))$, denoted as $L(x,\beta^{-1}(b))$. The probability of this is $Pr(B|X_i = x) = G_{(1)}(\beta^{-1}(b)|x) f_{(1)}(b)$.

Note there is a term $\frac{d\beta^{-1}(b)}{db}$ in $Pr(A|X_i = x)$, which is $\frac{dx}{db}$ when considering the symmetric bidding strategy $\beta(x) = b(x)$. It follows from the fact that if agent i increases the bid from b to $b+\Delta b$, it increases the probability of winning when Y_i is between $\beta^{-1}(b)$ and $\beta^{-1}(b+\Delta b)$ and thus $\frac{d\beta^{-1}(b)}{db}$ is needed to unify the scales.

In equilibrium all common-value bidders (including i) bid symmetrically, thus $x = b^{*-1}(b)$. Further denotes $H(x) = E(q|X_i = x, Y_i = x)$ and $L(x) = E(q|X_i = x, Y_i \le x)$, then the expected common value can be written as:

$$v(x,b,b^*(\cdot)) = \pi(x,b,b^*(\cdot))H(x) + (1 - \pi(x,b,b^*(\cdot)))L(x) \tag{1}$$

in which, $\pi(x,b,b^*(\cdot)) = \frac{Pr(A|X_i = x)}{Pr(A \cup B|X_i = x)}$. Note by strictly positive correlations, $H(x) > L(x)$ for all $x > \underline{x}$ and $H(\underline{x}) = L(\underline{x})$.

The following proposition describes the equilibrium:

Proposition 1. *The monotone pure-strategy equilibrium exists. In equilibrium, the private-value bidders bid their true private values, and the common-value bidders' bidding strategy $b^*(x)$ satisfies the pivotal condition $b^*(x) = v(x,b^*(x),b^*(\cdot))$, i.e. the solution to the following ODE:*

$$\frac{db}{dx} = \frac{(M-1)g(x|x)F(b)(H(x)-b)}{NG(x|x)f(b)(b-L(x))} \tag{2}$$

with the boundary condition $b(\underline{x}) = H(\underline{x}) = L(\underline{x})$ and $b'(\underline{x}) = \alpha L'(\underline{x}) + (1-\alpha)H'(\underline{x})$[6].

The proof contains the following two parts:

(1) Existence of an increasing solution
For all $x > \underline{x}$, since $L(x) < H(x)$, Eq. (2) suggests $lim_{b \to H(x)-} b'(x) = 0$ and $lim_{b \to L(x)+} b'(x) = +\infty$. Thus as $b(x)$ goes closely to $H(x)$, it stops increasing; and as $b(x)$ goes closely to $L(x)$, it increases dramatically. Intuitively (see Fig. 1 for the schematic diagram), $b(x)$ always lies in the area between the boundaries $L(x)$ and $H(x)$ once in it, and never reaches either boundary after $\underline{x}$. Notice $b'(\underline{x}) \in (L'(\underline{x}), H'(\underline{x})]$ suggests that $b(x)$ goes into the inner part of above area when $x > \underline{x}$. Then because of the continuity of RHS of Eq. (2), there exists a solution $b^*(\cdot)$ to Eq. (2) s.t. $L(x) < b^*(x) < H(x)$, for all $x \in (\underline{x}, \overline{x}]$[7].
Further, since $g(x|x), G(x|x), f(b), F(b) > 0$ for $x \in (\underline{x}, \overline{x}]$, we have $b^*(x) > 0$ for all $x \in [\underline{x}, \overline{x}]$ (recall $b^{*'}(\underline{x}) > L'(\underline{x}) \ge 0$). Thus $b^*(\cdot)$ is increasing.

(2) Global equilibrium
Here we verify the solution $b^*(x)$ is indeed a global equilibrium. From the common-value bidders' payoff function, we only need to show $v(x,b,b^*(\cdot)) - b > 0$ when $b < b^*(x)$, and $v(x,b,b^*(\cdot)) - b < 0$ when $b > b^*(x)$.
When $b < b^*(x)$, there exists $x' < x$ s.t. $b = b^*(x')$, as long as b is in the bidding range[8] since $b^*(x)$ strictly increases in x. We know

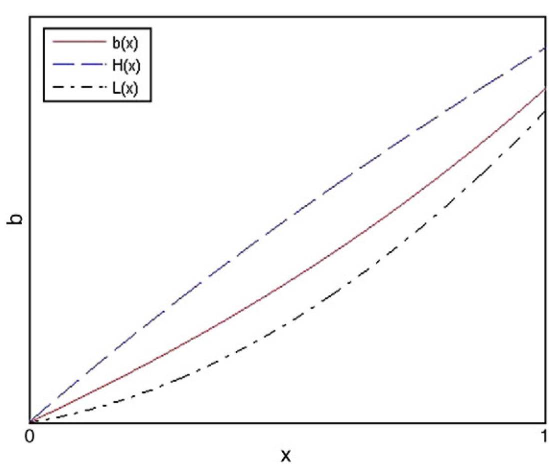

Fig. 1. Schematic diagram for the solution.

[2] It follows from the fact that usually the common-value bidders resell the item such that the common value should take all possible private values.

[3] Note that this assumption is easy to satisfy such as when the distribution of the signals conditional on the common value $h(x_i|q)$ are i.i.d.

[4] $f(x) \in C_1$, if and only if $f'(x)$ is continuous.

[5] $h_{(1)}$ denotes the distribution of the largest variable in a group. When n i.i.d. variables share the same density h, we have $h_{(1)} = nhH^{n-1}$.

[6] In which $\alpha = \frac{N}{N+M-1}$ if $H(\underline{x}) = \underline{z}$ and $\alpha = 0$ if $H(\underline{x}) > \underline{z}$. Moreover, we assume $H'(\underline{x}) > L'(\underline{x})$, by strictly affiliation almost all cases satisfy this assumption. See Appendix for detailed discussions.

[7] Peano Existence theorem guarantees the existence of the solution to Eq. (2), and Picard-Lindelof theorem further implies the uniqueness of solution for $x > \underline{x}$. The Appendix provides detailed discussions.

[8] Since $b^*(\cdot)$ is continuous, $b^*\left(\left[\underline{x}, \overline{x}\right]\right)$ is an interval. It is easy to show that no common-value agent wants to deviate to a bid out of this interval. So we only need to discuss the case where $b = b^*(x')$.