Economics Letters 108 (2010) 303–306

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Reserve prices in a dynamic auction when bidders are capacity-constrained

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ARTICLE INFO

Article history:
Received 29 January 2010
Received in revised form 7 May 2010
Accepted 20 May 2010
Available online 26 May 2010

Keywords:
Dynamic auction
Capacity constraint
Reserve price

JEL classification:
C73
D44

ABSTRACT

Allowing for a reserve price in a dynamic auction with capacity-constrained bidders changes the equilibrium in an unexpected way. The distribution of winning bids contains a mass point; several bidder types "bunch" at the reserve price.

1. Introduction

A frequent observation made by empirical researchers analyzing procurement auctions for road construction work is that construction firms experience capacity constraints, whereby the cost to a firm of adding another contract to its roster increases with its existing capacity utilization (Bajari and Ye, 2003; De Silva et al., 2002, 2003; Jofre-Bonet and Pesendorfer, 2003).[1] Since the cost distribution of the firm that wins the current auction worsens as a result of the capacity constraint, such firms face an intertemporal tradeoff in their profits: higher profits in the current period come at the cost of lower profits in future periods. Strategic forward-looking firms realize that they incur an opportunity cost or option value in the future by winning the current auction. The behavior of bidders in such auction settings has been theoretically analyzed by Grimm (2007), Jeitschko and Wolfstetter (2002), Jofre-Bonet and Pesendorfer (2006), and Saini (2009). The typical structure of equilibrium strategies is one where a bidder's bid equals the bidder's option value plus the amount that it would have bid in a static one-shot auction. Since the option value is just another markup over cost, bids increase with costs. However, none of the existing models consider the case where–as is common practice in procurement auctions–the procurer may impose a reserve price so that bids higher than the reserve price are rejected. Moreover, the structure of the equilibrium in these models does not allow for a reserve price.[2]

In this paper, we solve for the equilibrium bidding strategies of n capacity-constrained bidders in a sequence of two auctions where the procurer imposes a reserve price in the first period auction. Allowing for a reserve price changes the equilibrium in an unexpected way. We find that the equilibrium bidding strategies are no longer strictly increasing over the range of costs for which a bidder wins with a positive probability. More specifically, there exists an interval of a bidder's costs over which it always bids the reserve price, thus winning the auction with positive probability. Therefore, there exists a mass point in the distribution of winning bids at the reserve price. As usual, we also find a cutoff cost level such that firms with costs higher than the cutoff choose to drop out of the first auction by bidding some amount above the reserve price.

While we present our analysis in the context of a procurement auction, our analysis applies to a more general class of sequential auction models. For instance, an intertemporal linkage analogous to our model is possible in art auctions. An art collector interested in acquiring a series of paintings by a particular painter might find that winning one painting increases her valuation in future auctions for paintings from the same series. Our analysis predicts that if the

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[1] Here capacity utilization is defined as the ratio of the firm's outstanding work commitments to its size. The idea underlying this effect is that since a firm's capacity tends to stay fixed in the short term, as its work commitments increase, it must augment its capacity by paying overtime wages, renting additional equipment, moving scarce equipment around from site to site, and using less productive (perhaps older) equipment, all of which typically lead to higher unit costs.

[2] Suppose there are two sequential auctions. Starting from symmetry, in the absence of reserve prices, there is only one possible configuration of asymmetry in the second auction: someone wins the first auction and everyone else loses. However, in the presence of reserve prices, it is also possible that no one wins the first auction. This makes the bidding decision in the first auction more complicated than simply adding the option value to static bids.

0165-1765/$ – see front matter
doi:10.1016/j.econlet.2010.05.024

auctioneer imposes a reserve price in this setting, several bidder types will “bunch” at the reserve price. In addition to theorists, this result should also be of interest to researchers interested in recovering the underlying value distributions of bidders from their observed bid distributions in dynamic auction markets.

2. A model with reserve prices

A procurer auctions off two identical projects using a sequence of two first-price procurement auctions. There are n risk-neutral bidders, each of whom wishes to win both contracts. The costs of the bidders are independent private values. In the first auction, all bidders draw their cost for the project from the interval $[\underline{c},\overline{c}]$ according to a continuously differentiable probability distribution $F(c)$, with density $f(c)$ that is bounded away from 0 over the support. The auctioneer imposes a reserve price of R in the first auction.[3] In each round the bidders simultaneously submit sealed bids, and the contract is awarded to the lowest bidder. If there is more than one bidder at the lowest bid (which happens in equilibrium at R with positive probability), then the procurer awards the contract to each bidder with equal probability. A bidder can choose to drop out of the first auction by bidding an amount $OUT>R$.

Due to being capacity-constrained, the winning bidder experiences a first order stochastic dominance shift in its cost distribution on winning the first auction. As a result, the second auction becomes a one-shot asymmetric auction with one ‘weak,’ and $n-1$ identical ‘strong’ bidders. Standard results from the theory of static asymmetric auctions allow us to make the following abstraction regarding the profits in the second period auction (Maskin and Riley, 2000): if there is a winner in the first auction, then it gets a payoff of π_w in the second auction, and the remaining $n-1$ bidders get a payoff of π_l in the second auction, where $\pi_w<\pi_l$. If no one wins in the first period auction, all bidders get an expected payoff of π_0 in the second auction, where $\pi_w<\pi_0<\pi_l$. The idea is that the winning bidder's payoff in the second auction π_w, is lower than what its payoff would have been (π_0) if no one had won the first auction. For a similar reason, the second period payoff of a losing bidder π_l, is greater than π_0. Making this abstraction allows us to avoid the needless complication of describing the straightforward bidding strategies and profit relationships in the second period auction.

We consider the case where the reserve price is binding, that is, $R-(\pi_l-\pi_w)<c$. Otherwise, the reserve price does not affect behavior in the auction. We will explain the implication of this assumption below.

3. Equilibrium

Our main result is solving for a Perfect Bayesian Nash Equilibrium (PBNE) of this auction. While the formal statement of the result is summarized in Proposition 1, we now describe the structure of the equilibrium in words. Let $c_{j\neq i}^{min}$ denote the lowest cost draw among the rivals of bidder i in the first auction. The equilibrium bidding strategy in Eq. (1) can be described in three parts.

1. Over the cost interval $[\underline{c}, R-(\pi_l-\pi_w)]$, the players use monotonically increasing strategies. Each cost type's bid is the sum of two components. The first component is the expected minimum of $R-(\pi_l-\pi_w)$ and $c_{j\neq i}^{min}$ conditional on $c_{j\neq i}^{min}$ being higher than the bidder's cost type. This is a standard static bid. The second component $(\pi_l-\pi_w)$ is the option value of the bidder in the case where it is the winner in the first auction.
2. Interestingly, all cost types in the interval $[R-(\pi_l-\pi_w), c^*]$ bid the reserve price R; here c^* is as defined in Proposition 1. Thus, there is a mass point in the distribution of equilibrium bids at R. Another interesting feature of bidding over this cost range is that the bidders bid R in spite of the fact that by doing so they will not fully recover $c+(\pi_l-\pi_w)$, their cost draw plus the option value contingent on winning. This is so because with positive probability, the profit of the losing bidder is π_0 instead of π_l. Since $\pi_0<\pi_l$, the expected payoff upon losing the auction is lower.
3. All cost types higher than c^* drop out of the first auction by bidding $OUT>R$.

Proposition 1. *Each bidder bidding according to the following symmetric bidding strategy in the first auction constitutes a PBNE of the two-period auction.*

$$b(c)=\begin{cases}\mathbb{E}[\min\{R-(\pi_l-\pi_w),c_{j\neq i}^{\min}\}\,|\,c<c_{j\neq i}^{\min}]+(\pi_l-\pi_w) & \text{for } c\in[\underline{c},R-(\pi_l-\pi_w)]\\ R & \text{for } c\in[R-(\pi_l-\pi_w),c^*]\\ \text{OUT} & \text{for } c\in[c^*,\overline{c}].\end{cases}\tag{1}$$

where c^* solves

$$(R-c^*-(\pi_l-\pi_w))\sum_{k=0}^{n-1}\frac{1}{n-k}C_k^{n-1}[1-F(c^*)]^k[F(c^*)-F(R-(\pi_l-\pi_w))]^{n-1-k}$$
$$=(\pi_0-\pi_l)(1-F(c^*))^{n-1}.$$

Proof. We will show that the bidding strategy in Eq. (1) is optimal for a bidder given that the other bidders are playing the same strategy.

We will first write down an expression for a bidder's profits upon following this strategy. Since all of them bid R, the winning probability of cost types in the interval $[R-(\pi_l-\pi_w), c^*]$ is given by:

$$P_{win}(R;c^*)=\sum_{k=0}^{n-1}\frac{1}{n-k}C_k^{n-1}[1-F(c^*)]^k[F(c^*)-F(R-(\pi_l-\pi_w))]^{n-1-k}.$$

Here, $\frac{1}{n-k}C_k^{n-1}[1-F(c^*)]^k[F(c^*)-F(R-(\pi_l-\pi_w))]^{n-1-k}$ is the probability that k of the other $n-1$ bidders drop out of the first auction, and the remaining $n-k-1$ bidders draw a cost in the interval $[R-(\pi_l-\pi_w), c^*]$, causing them to bid R, which causes the procurer to award the project to each of the $n-k$ bidders, who bid R, with probability $\frac{1}{n-k}$ (tie-breaks). Given the strategies in Eq. (1), the expected profit of a bidder that gets a cost draw of c, and bids $b(c)$, is given by:

$$\Pi(b(c);c)=\begin{cases}(b(c)-c+\pi_w)(1-F(c))^{n-1}+\pi_l(1-(1-F(c))^{n-1}) & \text{if } b(c)<R\\ (R-c+\pi_w)P_{win}(R;c^*)+\pi_l(1-P_{win}(R;c^*)) & \text{if } b(c)=R\\ \pi_0(1-F(c^*))^{n-1}+\pi_l(1-(1-F(c^*))^{n-1}) & \text{if } b(c)=OUT>R.\end{cases}\tag{2}$$

If the bidder chooses to participate in the first auction and it bids less than R, then $(1-F(c))^{n-1}$ is the probability that the bidder wins the auction. If the bidder chooses to participate in the first auction with a bid of R, then $P_{win}(R;\,c^*)$ is the probability that it wins the auction. If the bidder drops out of the auction, then $(1-F(c^*))^{n-1}$ is the probability that every other bidder drops out as well, in which case the bidder gets a profit of π_0. The expression $1-(1-F(c^*))^{n-1}$ represents the probability that at least one of the rival bidders stays in the auction, in which case the bidder's payoff is π_l. The expression in Eq. (2) simplifies to:

$$\Pi(b(c);c)=\begin{cases}(b(c)-c-(\pi_l-\pi_w))(1-F(c))^{n-1}+\pi_l & \text{if } b(c)<R\\ (R-c-(\pi_l-\pi_w))P_{win}(R;c^*)+\pi_l & \text{if } b(c)=R\\ (\pi_0-\pi_l)(1-F(c^*))^{n-1}+\pi_l & \text{if } b(c)=OUT>R.\end{cases}\tag{3}$$

[3] The case where the procurer imposes a reserve price in the second auction as well is straightforward, since then the second auction becomes a simple static auction with a reserve price.