Economics Letters 108 (2010) 163–166

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

# International cross-holdings of bonds in a two-good DSGE model

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ARTICLE INFO

*Article history:*
Received 30 April 2009
Received in revised form 28 April 2010
Accepted 5 May 2010
Available online 12 May 2010

*Keywords:*
International diversification
Portfolio choice
Open economy macroeconomics
DSGE models

*JEL classification:*
F36
F41
G11

ABSTRACT

I solve for equilibrium portfolios in a two-country, two-good dynamic stochastic general equilibrium (DSGE) model where the only traded assets are locally-denominated real bonds. Unless the elasticity of substitution between goods is exceptionally low, the model predicts that each country will hold a short position in foreign bonds.



## 1. Introduction

Evidence suggests that cross-border holdings of bonds are large. For example, at the end of 2007, foreign holdings of U.S. corporate bonds amounted to 28% of the outstanding value of those bonds, and foreign holdings of U.S. Treasuries were 48% of their outstanding value (Federal Reserve Flow of Funds). Large foreign holdings of sovereign domestic debt are also prominent in the U.K. (32%, U.K. Debt Management Office), France (60%, Agence France Trésor) and other OECD countries. This paper asks a simple question: can a two-country, two-good equilibrium endowment model predict positive cross-holdings of bonds? If the elasticity of substitution between goods is sufficiently low, the answer is yes. However, the cutoff elasticity is at the lower end of estimates reported in the literature. For higher elasticities, the model predicts short foreign bond positions, which appear counterfactual for most advanced economies.

Most theoretical work on international diversification has focused on the "equity home bias puzzle": open economy macro models tend to predict much more cross-country diversification in equities than is observed in the data (see, e.g., Baxter and Jermann, 1997). A number of recent papers introduce bonds and equities together: Engel and Matsumoto (2009), Pavlova and Rigobon (2007), Coeurdacier et al. (2007) and Coeurdacier and Gourinchas (2008). However, all of these studies have more than one kind of shock in order to avoid portfolio indeterminacy. Furthermore, most of these models introduce bonds in order to improve the predictions for *equity* portfolios, rather than to study debt portfolios per se. Instead, I focus explicitly on bond portfolios in the simplest possible two-good model — one where the only shocks are to endowments.

## 2. Evidence on foreign bond positions

Lane and Shambaugh (2010) offer a framework for evaluating the foreign currency exposure in a country's balance sheet. They compute a country's foreign currency exposure in debt instruments, *FXD*, as the difference between foreign currency debt assets and foreign currency debt liabilities, divided by the sum of all foreign debt assets and liabilities. Table 1, column 1, presents this metric for a sample of advanced and emerging economies in 2004. Japan, China, India and Russia have long positions in foreign currency bonds; most other countries have short positions. However, as emphasized by Lane and Shambaugh (2010), *FXD* is driven primarily by a country's overall indebtedness, regardless of currency. Columns 1 and 2 of Table 1 demonstrate this forcefully: with the possible exception of the United Kingdom, all countries with positive (negative) net foreign asset positions in debt instruments (*NFAD*) also have positive (negative) foreign currency exposure.

Since the symmetric model that I analyze features zero net foreign assets in the steady-state, it is useful to consider an empirical measure of bond positions that abstracts from overall indebtedness. Again following Lane and Shambaugh (2010), I compute the "centered"

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doi:10.1016/j.econlet.2010.05.002

**Table 1**
Foreign currency exposure in debt instruments for selected countries, 2004. *FXD* is the difference between foreign currency debt assets and foreign currency debt liabilities, divided by the sum of all foreign debt assets and liabilities. *NFAD* is the net foreign asset position in debt instruments, divided by the sum of all foreign debt assets and liabilities. $FXD^0 = FXD - NFAD$.
Source: Author's calculations based on data from Lane and Shambaugh (2010) and Lane and Milesi-Ferretti (2007).

| Country | *FXD* | *NFAD* | $FXD^0$ |
|---|---|---|---|
| United States | −0.03 | −0.31 | 0.28 |
| United Kingdom | 0.00 | −0.06 | 0.06 |
| France | −0.06 | −0.04 | −0.02 |
| Germany | −0.05 | −0.02 | −0.04 |
| Japan | 0.39 | 0.38 | 0.01 |
| Canada | −0.23 | −0.34 | 0.11 |
| Italy | −0.05 | −0.20 | 0.15 |
| China | 0.58 | 0.58 | 0.00 |
| India | 0.10 | 0.10 | 0.00 |
| Brazil | −0.41 | −0.41 | 0.00 |
| Russia | 0.17 | 0.17 | 0.00 |

foreign currency exposure $FXD^0$ as the difference between *FXD* and *NFAD*. Conceptually, a positive value for $FXD^0$ indicates that a country would be long in foreign currency debt instruments if it had a zero net foreign asset position, holding the currency composition of assets and liabilities unchanged. Column 3 of Table 1 presents values for $FXD^0$. Except for France and Germany, the advanced countries in the sample have positive centered positions in foreign currency.[1] I interpret this as evidence that, abstracting from overall indebtedness, advanced countries tend to be long in foreign bonds.

## 3. The model

The model economy consists of two countries, home (H) and foreign (F). Each country features a "Lucas tree" that delivers a stochastic endowment of a country-specific good, $Y_t^i$, with $i \in \{H,F\}$. Country endowments (in logs) are assumed to follow a joint AR(1) process:

$$log\,(Y_t^i) = \rho\, log\,(Y_{t-1}^i) + \varepsilon_t^i \tag{1}$$

where $0 \le \rho < 1$ and $\epsilon_t \equiv (\epsilon_t^H, \epsilon_t^F)$ is a vector of zero-mean i.i.d. shocks with variance-covariance matrix $\Sigma$. These endowment shocks are the only source of uncertainty in the model.

Each country is populated with a continuum of identical households of mass 1. Households in country $i$ have preferences over a country-specific composite consumption good:

$$E_t\left\{\sum_{j=0}^{\infty} \beta^j \left[\frac{(C_{t+j}^i)^{1-\gamma}}{1-\gamma}\right]\right\} \tag{2}$$

where $0<\beta<1$ is the subjective discount factor and $\gamma>0$ is the (constant) coefficient of relative risk aversion. $C_t^i$ denotes country $i$'s consumption of its composite consumption good, which is a CES aggregate of home and foreign endowment goods:

$$C_t^i = \left[\lambda^{\frac{1}{\phi}}\left(C_t^{i,i}\right)^{\frac{\phi-1}{\phi}} + (1-\lambda)^{\frac{1}{\phi}}\left(C_t^{i,j}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, i \ne j \tag{3}$$

Here $C_t^{i,j}$ denotes country $i$'s consumption of endowment good $j$. $\lambda \in (0, 1)$ is the weight that households place on their own country's endowment good, and $\phi>0$ is the elasticity of substitution between H and F endowment goods.

There is no money in the model; all variables are real. Let $P_t^i$ denote the price of endowment good $i$ in terms of a numeraire (to be specified shortly). The consumer price index in country $i$, $P_{C,t}^i$, is:

$$P_{C,t}^i = \left[\lambda(P_t^i)^{1-\phi} + (1-\lambda)(P_t^j)^{1-\phi}\right]^{\frac{1}{1-\phi}}, i \ne j \tag{4}$$

Let the numeraire be a *world price index*:

$$\left(P_{C,t}^H\right)^{\frac{1}{2}}\left(P_{C,t}^F\right)^{\frac{1}{2}} = 1 \tag{5}$$

Define the terms of trade $TOT_t$ and the real exchange rate $RER_t$ as follows:

$$TOT_t = \frac{P_t^H}{P_t^F} \tag{6}$$

$$RER_t = \frac{P_{C,t}^H}{P_{C,t}^F} \tag{7}$$

Consider an environment in which the only traded assets are two real, infinitely-lived, locally-denominated bonds ("consols"). The home (foreign) bond offers a stream of constant payoffs of home (foreign) endowment goods. Let $P_{B,t}^i$ denote the price of the bond that delivers good $i$. Returns are given as follows:

$$R_t^i = \frac{P_{B,t}^i + P_t^i}{P_{B,t-1}^i} \tag{8}$$

Let $A_{t-1}^{i,j}$ denote country $i$'s holdings of the $j$-good bond at the end of period $t-1$, to be carried into period $t$. Note that asset holdings, asset prices and asset returns are all expressed in terms of the numeraire. Let $W_t^i \equiv A_t^{i,H} + A_t^{i,F}$ denote country $i$'s financial wealth at the end of period $t$. Following Devereux and Sutherland (2009), rewrite country $i$'s budget constraint as follows:

$$W_t^i = W_{t-1}^i R_t^F + A_{t-1}^{i,H}\left(R_t^H - R_t^F\right) + P_t^i Y_t^i - P_{C,t}^i C_t^i \tag{9}$$

A representative household in country $i$ maximizes Eq. (2) subject to Eqs. (3) and (9), taking prices as given.

I assume that bonds are in zero net supply:

$$A_t^{H,H} + A_t^{F,H} = 0 \tag{10}$$

$$A_t^{F,F} + A_t^{H,F} = 0 \tag{11}$$

The market-clearing conditions for goods are as follows:

$$C_t^{H,H} + C_t^{F,H} = Y_t^H \tag{12}$$

$$C_t^{F,F} + C_t^{H,F} = Y_t^F \tag{13}$$

An *equilibrium* is a sequence of stage-contingent values for consumption, bond holdings, goods prices, and bond prices such that all households behave optimally, taking prices as given, and goods and bond markets clear.

Following Coeurdacier et al. (2007), I further assume that $\gamma>1$ (households are more risk-averse than log investors) and $1/2<\lambda<1$ (countries exhibit consumption home bias). These assumptions are not necessary to solve the model, but they do simplify the interpretation of the equilibrium portfolios.

[1] The centered positions for China, India, Brazil and Russia are zero because 100% of these countries' foreign (debt-based) assets and liabilities are in foreign currency.