



Bertrand competition with cost uncertainty

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ABSTRACT

We analyze the classical model of Bertrand competition in a homogeneous good market with constant marginal costs and uncertainty regarding rivals' costs. First, we show that there exists a mixed strategy Nash equilibrium under the conventional equal sharing rule. Second, we illustrate the result for the case of piecewise-affine market demand.

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1. Introduction

Up until the last twenty years the Bertrand model of price competition was associated with the striking result that if two firms compete by simultaneously setting prices, both firms have symmetric constant returns to scale cost functions, and the market demand possesses a finite choke-off price, then the unique Nash equilibrium is that both firms price at marginal cost and earn zero profits. This, of course, is the famous “Bertrand paradox”.¹ However, subsequent research has revealed that the Bertrand model of price competition has other notable results. Dastidar (1995) showed that if firms have strictly convex costs, consistent with decreasing returns production technology, then there exists a continuum of pure strategy price equilibria in which the firms earn strictly positive profits. This result also holds if firms play mixed strategies. Hoernig (2002) has shown then when firms have strictly convex costs then there exist different types of mixed strategies with firms placing probability mass on a finite number of prices, or playing mixed strategies with continuous supports.

When firms have increasing returns to scale cost functions, the literature indicates that the existence of pure strategy Bertrand

equilibrium is problematic.² Dastidar (2006) has shown that when firms have increasing returns to scale cost functions then there does not exist any pure strategy price equilibrium under the conventional equal sharing rule. However, it is well-known that if the sharing rule is altered so that a single firm is selected randomly from the set of firms tying at the minimum price to serve all the market demand, the so-called “winner-takes-all” sharing rule, then this restores the existence of pure strategy equilibrium (Vives, 1999, p.119). The existence of a mixed strategy equilibrium with increasing returns to scale costs has, up until recently, been an open question. In their survey of the literature Baye and Kovenock (2008) proved that there does not exist a mixed strategy equilibrium either.

An extension of the Bertrand model is to consider the case when firms have asymmetric costs. If both firms have constant marginal costs, and the high marginal cost is below the profit maximizing price for the low-cost firm, it has recently been shown that the price equilibrium has the low-cost firm setting price equal to the higher marginal cost, and the high-cost firm randomizing uniformly on an interval above (Blume, 2003).³ Related to asymmetric costs is research which analyzes the Bertrand model when costs are

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¹ If no finite choke-off price exists, and monopoly revenues are unbounded, then in addition to the pure strategy zero profit Nash equilibrium, there exist a continuum of atomless mixed strategy equilibria in which the firms earn positive profits (Kaplan and Wettstein, 2000).

² As Bertrand competition is a game with discontinuous payoffs, the Glicksberg–Fan theorem cannot be used to establish existence, and more recent existence results for discontinuous games, such as Dasgupta and Maskin (1986) and Reny (1999), are not applicable because payoffs are neither quasi concave, sum upper semicontinuous nor reciprocally upper semicontinuous.

³ For a long time this simple case of asymmetric costs did not have a well-defined solution. It was typical to assume the price space was discrete and the low-cost firm would price at some small unit, a minimum currency, below the high marginal cost. See, for example, Tirole (1988, p.211).

uncertain. Typically, it is assumed each firm knows their own cost type but does not know the cost type of their rivals. Spulber (1995) showed that if the cost function is parameterized, and the parameter drawn from a continuous probability distribution, then if firms are uncertain about their rivals' cost profiles, they price above marginal cost and make positive profits. Vives (1999, p.230) analyzed the general case where firms have constant marginal costs drawn from a continuous probability distribution with compact support, and showed by the log-supermodularity of the expected profit function that there always exists a Bayesian Nash equilibrium in which firms set prices contingent upon cost type. In the context of auctions, Hansen (1988) considered the case where two risk-neutral producers of a homogeneous good simultaneously bid to serve a market and have uncertain marginal costs. The optimal bidding strategy was characterized and a comparison with price-setting oligopoly models was provided.

However, there is a notable gap in the research. There are no equilibrium existence results for the classical Bertrand model when there is discrete cost uncertainty. A survey of textbook questions on market competition reveals that the problem of finding a Bayesian Nash equilibrium in the homogeneous good Cournot model with discrete cost uncertainty is a frequently encountered question (Mas-Colell et al., 1995, p.265).⁴ Bertrand competition with differentiated goods and discrete cost uncertainty is also a commonly encountered textbook problem (Tirole, 1988, p.362). But the case of homogeneous good Bertrand price competition, with constant returns to scale production technology, and discrete cost uncertainty, does not appear in any textbooks or in the professional literature. The aim here is to fill this gap. The main result (Proposition 2) is that there exists a mixed strategy equilibrium. The next section of the paper presents the model and result. The final section draws some conclusions regarding future research.

2. The model

Consider the market for a homogeneous good in which there are $N = \{1, \dots, n\}$, $n \geq 2$, firms which compete by simultaneously and independently setting prices. The market demand $D: \mathcal{P}_+ \rightarrow \mathcal{P}_+$ is C^2 and $\exists$ positive finite real numbers $P^{\max}, Q^{\max}$, satisfying $D(P^{\max}) = 0$ and $D(0) = Q^{\max}$. Also, $D'(P) < 0 \forall P \in (0, P^{\max})$. Firms' cost functions are derived from constant returns to scale production technology and take one of two forms: high or low. That is, $C_i(Q) = c_i Q$ with $i \in \{H, L\}$ and $0 < c_L < c_H < P^{\max}$. Each firm has probability θ of having marginal cost c_L and probability $1 - \theta$ of having marginal cost c_H with $\theta \in (0, 1)$. It is assumed each firm knows their own cost type but does not know the cost type of their rivals. Firms supply all the demand they face and maximize their expected profits.⁵ Ties at the lowest price are resolved by firms sharing the market demand equally. Define $\pi_i(P)$ to be the monopoly profit of a firm with cost type i , and $\pi_i(P, m)$ to be the shared profit of a firm with cost type i when it ties with $m - 1$ firms at the lowest price:

$$\pi_i(P) = (P - c_i)D(P) \quad (1)$$

$$\hat{\pi}_i(P, m) = \frac{1}{m}(P - c_i)D(P) \quad (2)$$

Finally, we assume that $\pi_i(P)$ and $\hat{\pi}_i(P, m)$ are strictly concave in price. All the assumptions employed here are standard specifications of the Bertrand model apart from the uncertainty regarding costs. In

the classical Bertrand model the firms have the same cost type and the price-setting game is a one of complete information. Here, each firm only knows their own cost type and the probability distribution over the possible cost types of their rivals. As a result, the price-setting game becomes a game of incomplete information. Let $P = [0, P^{\max}]$ denote the pure strategy price space, and let $\mathcal{G}$ denote the game in which firms simultaneously and independently set prices. In order to simplify the main result define the following:

$$P^* = \arg \max_{P \in \mathcal{P}} \pi_L(P)$$

$$\hat{P} = \min \{c_H, P^*\}$$

In words, P^* is the monopoly price for the low-cost firm, and $\hat{P}$ is the minimum of the high marginal cost and P^* .

Lemma 1. There exists a unique $\tilde{P} \in (c_L, \hat{P})$ s.t. $\pi_L(\tilde{P}) = (1 - \theta)^{n-1} \pi_L(\hat{P})$.

Proof. First, note that $0 < (1 - \theta)^{n-1} \pi_L(\hat{P}) < \pi_L(\hat{P})$. As the profit function is continuous, and $\pi_L(c_L) = 0$, the intermediate value theorem⁶ guarantees $\exists$ a $\tilde{P} \in (c_L, \hat{P})$ such that $\pi_L(\tilde{P}) = (1 - \theta)^{n-1} \pi_L(\hat{P})$. The strict concavity of the profit function implies $\pi_L'(P) > 0$ for all $P \in (c_L, \hat{P})$. This ensures that $\tilde{P}$ is unique. $\square$

Proposition 1. $\mathcal{G}$ does not possess a pure strategy Bayesian Nash equilibrium.

Proof. Start by assuming there does exist a pure strategy equilibrium. There are two cases to consider. The first case is when the firms play a symmetric pure strategy. The second case is when the firms play an asymmetric pure strategy.

Case 1. The firms play a symmetric pure strategy with the low-cost firms setting a price $P^L > c_L$ and the high-cost firms setting a price $P^H > c_H$, and suppose, without loss of generality, that $P^L < P^H$. That is, when low-cost firms are in the market, high-cost firms do not receive any demand. The expected payoff to the high-cost firm is $(1 - \theta)^{n-1} \hat{\pi}_H(P^H, n)$. As $\pi_H(P) > \hat{\pi}_H(P, n)$ for all $P \in (c_H, P^{\max})$ one firm could deviate to a price $P^H - \varepsilon > c_H$ and increase their expected profit. The possibility of undercutting means that in any equilibrium the high-cost firm must price at marginal cost: $P^H = c_H$. Now consider the low-cost firm. The expected profit is:

$$(1 - \theta)^{n-1} \pi_L(P^L) + \sum_{r=1}^{n-1} \binom{n-1}{r} \theta^r (1 - \theta)^{n-1-r} \hat{\pi}_L(P^L, r + 1)$$

which is strictly less than $\pi_L(P^L)$. This means one low-cost firm could deviate to a price $P^L - \varepsilon > c_L$ and increase its expected profit. As with the high-cost firm, the possibility of undercutting leads to the consideration of $P^L = c_L$ as a possible equilibrium. However, this cannot be an equilibrium because if $P^L = c_L$ then one firm could deviate to a price $P \in (c_L, c_H)$ and earn expected profit of $(1 - \theta)^{n-1} \pi_L(P) > 0$. Hence, there does not exist a symmetric pure strategy equilibrium.

Case 2. The firms play asymmetric pure strategies. The same reasoning as above means that the high-cost strategy must have at least two firms pricing at marginal cost, and all other firms either pricing at marginal cost or a higher price.⁷ However, as no low-cost firms want to tie at the same price suppose, without loss of generality, that the pricing strategies of the low-cost firms are $c_L \leq P_1^L < P_2^L < \dots <$

⁴ Although Einry et al. (2010) have shown that when firms have differential information regarding costs and/or demand, and there are a finite number of states, then a pure strategy Bayesian Cournot equilibrium may fail to exist.

⁵ This is what distinguishes Bertrand competition from Bertrand-Edgeworth competition. In Bertrand-Edgeworth competition firms may refuse to supply all the demand forthcoming at any price. For a succinct summary, see Vives (1999, Ch.5).

⁶ See, for example, Rudin (1976, p.93) Theorem 4.23.

⁷ This is the standard Bertrand equilibrium when there are two or more firms in the market.

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