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Regulating risk-averse producers: The case of complementary products

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ABSTRACT

This paper studies the impacts of producers' risk-aversion on the relative virtues of integrated production and component production, in the case of complementary products with independent cost realization.

1. Introduction

In multiproduct industry, a regulator often faces the problem of selecting a proper production form, for example, between *integrated production* and *component production*. Integrated production means a single producer supplies all products, while component production means different products are separately supplied by different producers. The relative virtues of these two production forms depend on specific industrial features and production technologies. If there is significant effect of technological economy of scope, then integrated production can dominate component production. On the other hand, if there is no such effect, it is shown in the case of independent products with correlated costs that, when the correlation is low, the regulator prefers integrated production, due to informational economy of scope under integrated production, but when the correlation is sufficiently high, component production is preferred instead, since the regulator can exploit the benefits of yardstick competition under component production (Dana, 1993; Armstrong, 1999; Armstrong & Sappington, 2007).

Gilbert and Riordan (1995, thereafter GR) study the problem of regulating complementary products with independent cost realizations. It is obvious that, in their setting, there is neither technological economy of scope nor the benefits of yardstick competition. In this case, they prove that the regulator prefers integrated production over component production, due to the benefit of informational economy (Baron & Besanko, 1992) of scope.

To our knowledge, the existing literature commonly assumes that the producers are risk-neutral. However, risk-averse producers seems a more realistic assumption in many cases. For example, a producer may worry about not only the expected value, but the volatility of his revenue streams. Does producers' risk-aversion have any impact on the relative virtues of integrated and component productions? This paper tries to answer this question in the case of complementary products.

Following the same setting as GR, we prove two important results. First, component production by risk-averse producers generates higher revenue than that by risk-neutral producers. This is because the aversion to outcome uncertainty relaxes the incentive constraints, and thus reduces the information-rent to the producers. Second, when the producers are risk-averse, the relative virtues of integrated and component production depend on the degree of risk aversion. Specifically, in the case of CRRA preference, we prove that component production generates higher expected revenue than integrated production iff the coefficient of risk-aversion is higher than some threshold.

The paper is organized as follows. Section 2 presents the model. Section 3 investigates the relative virtues of integrated and component productions under risk-aversion producers. And Section 4 is the conclusion.

2. The model

A regulator contracts for a final good made up of two complementary intermediate products. The cost of a unit of final good is the

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sum of the costs of a unit of each intermediate product. The regulator's problem is two folds, selecting a proper production form and designing optimal production contacts. Both products are produced at constant marginal costs, which are drawn independently from the same distribution

$$c_i = \begin{cases} \underline{c} & \Pr = p \\ \bar{c} & \Pr = 1-p \end{cases} \quad i = 1,2$$

and $\Delta c = \bar{c} - \underline{c} > 0$. The realization of c_i is observable only to the producer of product i. We also keep GR's assumption that when cost realizations $(c_1, c_2) = (\bar{c}, \bar{c})$, the production is not profitable.

For integrated production, the contract is a scheme of combination of transfers and outputs, $\{T(\hat{\mathbf{c}}), Q(\hat{\mathbf{c}})\}$, conditional on the producer's cost announcement $\hat{\mathbf{c}} = (\hat{c}_1, \hat{c}_2)$. $T(\hat{\mathbf{c}})$ is the transfer to the producer, and $Q(\hat{\mathbf{c}})$ is the output of the final good. The regulator is risk neutral, and his revenue is

$$W_j^I(\hat{\mathbf{c}}) = S(Q(\hat{\mathbf{c}})) - T(\hat{\mathbf{c}})$$

where $j = n, a$ denotes the cases of risk-neutral (n) and risk-averse (a) producers, and $S(\cdot)$ is increasing and strictly concave. When the producer is risk neutral, his vNM utility is just the amount of net monetary transfer, that is, $U(\hat{\mathbf{c}}|\mathbf{c}) = T(\hat{\mathbf{c}}) - (c_1 + c_2)Q(\hat{\mathbf{c}})$, where $\mathbf{c} = (c_1, c_2)$ is the true realization of the marginal costs. When the producer is risk averse, his vNM utility is

$$U(\hat{\mathbf{c}}|\mathbf{c}) = u[T(\hat{\mathbf{c}}) - (c_1 + c_2)Q(\hat{\mathbf{c}})]$$

and $u(\cdot)$ is a strictly concave and increasing function satisfying Inada conditions, with the normalization that $u(0) = 0$.

For component production, the contract is $\{t_1(\hat{\mathbf{c}}), t_2(\hat{\mathbf{c}}), q(\hat{\mathbf{c}})\}$, where t_i is the transfer to the producer of product i and $q(\cdot)$ is the output level. The regulator's revenue is thus

$$W_j^C(\hat{\mathbf{c}}) = S(q(\hat{\mathbf{c}})) - t_1(\hat{\mathbf{c}}) - t_2(\hat{\mathbf{c}}).$$

For risk-neutral producers, the vNM utility is $U_i(\hat{\mathbf{c}}|c_i) = t_i(\hat{\mathbf{c}}) - c_i q(\hat{\mathbf{c}})$ for $i = 1, 2$. For risk-averse producers, they are

$$U_i(\hat{\mathbf{c}}|c_i) = u[t_i(\hat{\mathbf{c}}) - c_i q(\hat{\mathbf{c}})] \; i = 1,2$$

We consider only symmetric equilibrium in our model, which implies that $q(\underline{c}, \bar{c}) = q(\bar{c}, \underline{c})$, $t_1(\underline{c}, c) = t_2(c, \underline{c})$ and $t_1(\bar{c}, c) = t_2(c, \bar{c})$ for $c = \underline{c}, \bar{c}$.

The timing of the game is as follows: (1)The regulator selects the production form and proposes the contract to the producer(s); (2) c_i is observed by relevant producer, and $(\hat{c}_1, \hat{c}_2)$ is report; (3) If $\hat{c} = (\bar{c}, \bar{c})$, the game is ended and each player receives a payoff of 0; otherwise, the project is implemented. (4) Production is finished and transfers are paid according to the contract.

3. Regulating risk-averse producers

For the case of risk-neutral producers, GR have shown under the same setting that the regulator prefers integrated production to component production. We will show that this is not necessarily true in the case of risk-averse producers.

3.1. Component production

We first introduce some new notations for output and net monetary transfer: $\underline{q} = q(\underline{c}, \underline{c})$, $\hat{q} = q(\underline{c}, \bar{c}) = q(\bar{c}, \underline{c})$; $\underline{m} = t_1(\underline{c}, \underline{c}) - \underline{c} \cdot \underline{q}$, $\hat{m} = t_1(\underline{c}, \bar{c}) - \underline{c} \cdot \hat{q}$, $\bar{m} = t_1(\bar{c}, \underline{c}) - \bar{c} \cdot \hat{q}$.[1] In equilibrium, the following interim incentive compatible (IC) conditions for producers should be satisfied

$$\underline{IC}(c_i = \underline{c}) pu(\underline{m}) + (1-p)u(\hat{m}) \geq pu(\bar{m} + \Delta c \cdot \hat{q}) \quad (1)$$

$$\overline{IC}(c_i = \bar{c}) pu(\bar{m}) \geq pu(\underline{m} - \Delta c \underline{q}) + (1-p)u(\hat{m} - \Delta c \hat{q}) \quad (2)$$

So should the following interim individual rationality (IR) conditions.

$$\underline{IR}(c_i = \underline{c}) \quad p \cdot u(\underline{m}) + (1-p) \cdot u(\hat{m}) \geq 0 \quad (3)$$

$$\overline{IR}(c_i = \bar{c}) \quad pu(\bar{m}) \geq 0 \quad (4)$$

And the regulator's ex ante expected revenue is

$$EW_a^C = p^2[S(\underline{q}) - 2\underline{c}\,\underline{q} - 2\underline{m}] + 2p(1-p)[S(\hat{q}) - (\underline{c} + \bar{c})\hat{q} - \hat{m} - \bar{m}]$$

and his objective is to maximize EW_a^C subject to the IC and IR constraints, that is

$$(\mathcal{P}_1) \begin{array}{ll} \max & EW_a^C \\ \text{s.t.} & (1)-(4) \end{array}$$

We need first to clarify which constraints are binding in $(\mathcal{P}_1)$, and the result is as below.

Lemma 1. *In the case of risk averse producers, only constraints (1) and (4) are binding in the optimality of $(\mathcal{P}_1)$.*

Proof. From (1), we have

$$\begin{aligned} pu(\underline{m}) + (1-p)u(\hat{m}) &\geq p \cdot \{u(\bar{m}) + u'(\bar{m} + \Delta c\hat{q}) \cdot \Delta c\hat{q}\} \\ &= p \cdot u(\bar{m}) + A \end{aligned}$$

where $u(\bar{m}) \geq 0$ and $A > 0$. Thus (4) is binding but (3) is not. From Eq. (2), we have

$$\begin{aligned} pu(\bar{m}) &\geq p\{u(\underline{m}) - u'(\underline{m} - \Delta c\underline{q}) \cdot \Delta c\underline{q}\} + (1-p)\{u(\hat{m}) - u'(\hat{m} - \Delta c\hat{q}) \cdot \Delta c\hat{q}\} \\ &= pu(\underline{m}) + (1-p)u(\hat{m}) - \Delta c\{p\underline{q}u'(\underline{m} - \Delta c\underline{q}) + (1-p)\hat{q} \cdot u'(\hat{m} - \Delta c\hat{q})\} \\ &= pu(\underline{m}) + (1-p)u(\hat{m}) - B \end{aligned}$$

From $u(\bar{m}) = 0$, we have $B \geq p \cdot u(\underline{m}) + (1-p) \cdot u(\hat{m}) \geq A$. The regulator wants to reduce the rent of the producers, and therefore (1) is binding but (2) is not. □

Given this result, we can easily write down the Lagrangian of $(\mathcal{P}_1)$, and characterizes the optimal contract for component production as below.

Proposition 2. *The optimal allocation of $(\mathcal{P}_1)$, $\{\underline{q}^{Ca}, \hat{q}^{Ca}; \underline{m}^{Ca}, \hat{m}^{Ca}, \bar{m}^{Ca}\}$, satisfies the following conditions*[2]:

$$S'(\underline{q}^{Ca}) = 2\underline{c} \quad (5)$$

$$S'(\hat{q}^{Ca}) = (\underline{c} + \bar{c}) + \frac{p}{1-p} \cdot \frac{u'(\Delta c\hat{q}^{Ca})}{u'(\underline{m}^{Ca})}\Delta c \quad (6)$$

$$u(\underline{m}^{Ca}) = pu(\Delta c\hat{q}^{Ca}) \quad (7)$$

$$\bar{m}^{Ca} = 0, \; \underline{m}^{Ca} = \hat{m}^{Ca}. \quad (8)$$

[1] In symmetric equilibrium, for the producer of product 2, we have $\underline{m} = t_2(\underline{c}, \underline{c}) - \underline{c} \cdot \underline{q}$, $\hat{m} = t_2(\bar{c}, \underline{c}) - \underline{c} \cdot q$, $\bar{m} = t_2(\underline{c}, \bar{c}) - \bar{c} \cdot \hat{q}$.

[2] The superscripts Ca and Cn mean “component production” by “risk-averse” and “risk-neutral” producers respectively.