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Effective contests

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ABSTRACT

We find that two-stage contests could be ineffective, namely, there is a higher chance of low-ability players participating (and winning) than high-ability players. However, imposing a fee on the winner can guarantee that the contest will be effective.

1. Introduction

In many contests, players first have to decide whether or not to enter a contest. Afterwards, if there is more than one entrant, they must compete to determine the winner. For example, these can occur in promotion contests, open calls for research, or political contests.

In this paper, we model this situation as a two-stage game where there is an entry stage and a contest stage. The cost of participating in the contest has two components: first, the entry costs and second, the expenditure used in the contest. We model the former as privately known and fixed and the latter as publically known and variable. Also, each player has a publically known ability, either high or low.

The timing of the model is as follows. In entry stage, the players engage each other. They indicate their interest in entering and every player learns the abilities of his potential opponents. Then, given his private cost of entry, he decides whether or not to participate in the contest stage. The players who decide to participate pay their entry costs. After this stage, all players incur their entry costs and learn who has entered. In the second stage, the players compete against each other in what we model as an asymmetric all-pay auction under complete information.[1] Each player chooses expenditure (effort) and the player with the highest expenditure wins the contest. Ability factors into the cost of expenditure. Those with higher ability have an easier time competing. Independent of success, all players bear the costs.

We find that our model has cutoff equilibria, where any player with an entry cost higher than the cutoff for his type (ability) will decide to stay out of the contest and any player with an entry cost lower than the cutoff for his type will decide to participate in the contest. We show that given these equilibrium entry decisions the contest may be ineffective; namely, the chance that a high-ability player will participate may be lower than the chance that a low-ability player will participate. Consequently, there may be a higher chance that the winner of the contest will be the low-ability player. We show that a designer can overcome this problem and guarantee that the contest will be effective by imposing a requirement (task or fee) to be paid by the winner of the contest.

2. The model

Consider n players competing in a contest for one prize. The players have the same value for winning the position (contest) which

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[1] In the economic literature, all-pay auctions are studied under complete information where the players' valuations for the object are common knowledge (see, for example, Hillman and Riley, 1989; Baye et al., 1993, 1996; Che and Gale, 1998; Kaplan et al., 2003) or under incomplete information where each player's valuation for the object is private information to that player and only the distribution of the players' valuations is common knowledge (see, for example, Amann and Leininger, 1996; Krishna and Morgan, 1997; Moldovanu and Sela, 2001, 2006).

is normalized to be 1. Player i's ability, $\alpha_i \geq 0$, is common knowledge. Assume that there are n_1 players with high ability of α_1 and n_2 players with a low ability of $\alpha_2 < \alpha_1$.[2] Participating in the contest generates a (sunk) cost c_i/α_i for player i, where c_i is the entry cost which is private information and is drawn independently from the cumulative distribution function F which is on the interval $[\underline{c}, \bar{c}]$ where $0 \leq \underline{c} < \min\ \alpha_i$. We assume that F is continuously differentiable with $F(\underline{c}) = 0$ and is common knowledge.[3] In the first stage, all the players are engaged, they learn the valuations of their opponents and each one decides whether to stay out or participate in the second stage of the contest. The players who decide to participate pay their entry costs. Then, in the second stage, these players see who else has decided to participate and compete in an all-pay auction under complete information such that the player with the highest expenditure x_i wins the contest, while all the players pay their cost of expenditure, which is $\frac{x_i}{a_i}$ (higher ability players have an easier time putting forth an effort). Thus, if player i decides to participate at the second stage of the contest, pays his entry cost c_i, spends an expenditure of x_i and wins the contest, then his payoff is given by $1 - \frac{(x_i + c_i)}{\alpha_i}$. On the other hand, if he does not win the contest his payoff is given by $-\frac{(x_i + c_i)}{\alpha_i}$.

3. Equilibrium

In our model there are frequently trivial equilibria strategies in which one of the players decides to always participate independent of his entry cost, and all the other players decide to stay out of the contest in the second stage. In order to prevent such equilibrium strategies (when n_1, $n_2 > 1$) we assume that players of the same type (same α) follow the same strategy. We say that an equilibrium is type-symmetric if all players of the same type follow the same strategy.

In the second stage the players compete in the all-pay auction where the players' abilities are common knowledge. If there is only one entrant in the second stage, he will bid zero and win. If there is more than one entrant, there are three cases that need to be examined. Let us denote e_i for the number of entrants of type i. By Baye et al. (1996), we have the following type-symmetric equilibrium in the second stage:

Case 1. There are two or more entrants with low abilities (type 2) only.

Then, these players randomize on the interval $[0, \alpha_2]$ according to their expenditure cumulative distribution functions $F_2(x)$, which is given by the indifference condition:

$$\alpha_2 F_2^{e_2 - 1}(x) - x = 0. \tag{1}$$

Thus, each player's expenditure is distributed according to $F_2(x) = \left(\frac{x}{\alpha_2}\right)^{\frac{1}{e_2 - 1}}$. Total expenditure is $e_2 \int_0^{\alpha_2} x dF_2(x) = \alpha_2$ and the expected payoff of every player is $u_2 = 0$.

Case 2. There are $e_1 \geq 2$ entrants with high abilities (type 1) and any number of entrants with low abilities.

In this case all the players of type 2 stay out and the players of type 1 enter in the second stage. These players randomize on the interval $[0, \alpha_1]$ according to their expenditure cumulative distribution functions $F_1(x)$, which is given by the indifference condition:

$$\alpha_1 F_1^{e_1 - 1}(x) - x = 0. \tag{2}$$

Thus, players' expenditure is distributed according to $F_1(x) = \left(\frac{x}{\alpha_1}\right)^{\frac{1}{e1 - 1}}$. The total expected expenditure is $e_1 \int_0^{\alpha_1} x dF_1(x) = \alpha_1$ and the expected payoff of every player is $u_1 = 0$.

Case 3. There is only one entrant with high ability and $e_2 \geq 1$ entrants with low abilities.

Then, the players randomize on the interval $[0, \alpha_2]$ according to their expenditure cumulative distribution functions, $F_1(x)$ and $F_2(x)$, which are given by the indifference conditions:

$$\begin{aligned} &\alpha_1 F_2^{e_2}(x) - x = \alpha_1 - \alpha_2, \\ &\alpha_2 F_1(x) - x = 0. \end{aligned} \tag{3}$$

Thus, type 1's expenditure is distributed according to $F_1(x) = \frac{x}{\alpha_1}$, while type 2's expenditure is distributed according to $F_2(x) = \left(\frac{x + \alpha_1 - \alpha_2}{\alpha_1}\right)^{\frac{1}{e_2}}$. The total expected expenditure is $\int_0^{\alpha_2} x dF_1(x) + e_2 \int_0^{\alpha_2} x dF_2(x) = \frac{\alpha_2 + 3e_2\alpha_2 + 2e_2^2(\alpha_1 - \alpha_2)\left(\left(1 - \frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{e_2}} - 1\right)}{2(e_2 + 1)}$, and the respective expected payoffs are $u_1 = \alpha_1 - \alpha_2$ and $u_2 = 0$.

Now, given the analysis of the players' behavior in the second stage of the contest, we can analyze their entry decisions in the first stage. In the first stage, n_1 players with ability of α_1 and n_2 players with ability of α_2 are engaged and each of them decides whether to participate or not, and those who decide to participate pay their private entry costs. Denote by $d_i(c)$ the entry decision (the probability of entering) if one has entry cost c and ability $\alpha_i > 0$.

Proposition 1. *The entry decision (the probability of entering) of a player with cost c_i and ability $\alpha_i > 0$ in the first stage is*

$$d_i(c) = \begin{cases} 1 \text{ if } c \leq c_i^*, \\ 0 \text{ if } c > c_i^* \end{cases}$$

where the equilibrium cutoffs c_i^*, $i = 1,2$ are given by[4]

$$c_1^* = (\alpha_1 - \alpha_2)(1 - F(c_1^*))^{n_1 - 1} + \alpha_2(1 - F(c_2^*))^{n_2}(1 - F(c_1^*))^{n_1 - 1}, \tag{4}$$

$$c_2^* = \alpha_2(1 - F(c_1^*))^{n_1}(1 - F(c_2^*))^{n_2 - 1}. \tag{5}$$

In the symmetric case where $\alpha = \alpha_1 = \alpha_2$ and n is the total number of players, the symmetric entry decision is given by

$$d_i(c) = \begin{cases} 1 \text{ if } c \leq c^*, \\ 0 \text{ if } c > c^* \end{cases}$$

where the equilibrium cutoff $c^* > 0$ is the solution of[5]

$$c^* = \alpha(1 - F(c^*))^{n-1}. \tag{6}$$

Proof. See Appendix A. □

The entry decision described by Proposition 1 is such that any player with ability α_i and an entry cost higher than the equilibrium cutoff c_i^* will stay out of the contest and any player with ability α_i and an entry cost lower than the equilibrium cutoff c_i^* will participate in the second stage of the contest.

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[2] For simplicity, we assume two types of abilities. Our results can be generalized to the case with any number of types.

[3] To avoid a trivial solution assume that $F(\alpha_2) > 0$ (there is a chance that player i has a cost lower than α_2).

[4] Obviously, this equilibrium is for n_1, $n_2 \geq 1$. If $n_1 \geq 2$, $n_2 \geq 2$ and $\underline{c} = 0$, then any type-symmetric equilibrium must be interior. If $n_1 = 1$ or $n_2 = 1$ the type-symmetric equilibrium can be non-interior with $c_1^* \geq \bar{c}$, $c_2^* \leq \underline{c}$ or $c_2^* \geq \bar{c}$, $\underline{c} < c_1^* < \bar{c}$ (and for $\underline{c} > v1 - v2$, non-interior with $c_2^* \geq \bar{c}$, $c_1^* \leq \underline{c}$). A cutoff $c_i > \bar{c}$ implies that everyone of type i would enter and a cutoff $c_i < \underline{c}$ implies that everyone of type i stays out.

[5] For the symmetric case, any symmetric equilibrium is interior.