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The Error-in-Rejection Probability of meta-analytic panel tests

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ABSTRACT

The puzzling Monte Carlo finding that the size distortion of meta-analytic panel unit root tests increases with the number of panel series is explained as the cumulative effect of arbitrarily small size distortions in the time series tests composing the panel test.

1. Introduction

Meta-analysis (see Hedges and Olkin, 1985) is a useful tool to efficiently combine related information. In recent years, the meta-analytic testing approach has been fruitfully applied to nonstationary panels: Consider the testing problem on the panel as consisting of N testing problems for each unit of the panel. That is, conduct N separate time series tests and obtain the corresponding p-values of the test statistics. Then, combine the p-values of the N tests into a single panel test statistic. Among others, Maddala and Wu (1999), Choi (2001) and Phillips and Sul (2003) propose meta-analytic panel unit root and cointegration tests. The tests are intuitive, relatively easy to compute and allow for a considerable amount of heterogeneity in the panel.

Via Monte Carlo experiments, the above-cited authors show that their meta-analytic tests can be substantially more powerful than separate time series tests on each unit. Disturbingly, however, Choi (2001) and Hlouskova and Wagner (2006), inter alia, find the Error-in-Rejection Probability (ERP) (or, synonymously, size distortion) to be increasing in N. That is, the (absolute) difference between the estimated rejection probability $R(\alpha, N)$ and the nominal significance level α, $\text{ERP}_N(\alpha) := |R(\alpha, N) - \alpha|$, gets larger with N. A priori, this finding is counterintuitive, since more information should improve the performance of the panel tests.

We argue that this behavior may be explained as the cumulative effect of arbitrarily small ERPs in the underlying time series test statistics composing the panel test statistics. Under a simple H_0, assuming continuous distribution functions of the test statistics, p-values of test statistics should be distributed uniformly on the unit interval, denoted $\mathcal{U}[0, 1]$. The analytical and simulation evidence reported in the following sections corroborate our conjecture.

2. The *p*-value combination test

We briefly review the p-value combination test whose ERP is investigated subsequently. We discuss the example of a panel unit root test. Denote by p_i the marginal significance level, or p-value, of a time series unit root test applied to the ith unit of the panel. Let θ_{i,T_i} be a unit root test statistic on unit i for a sample size of T_i. Let F_{T_i} denote the null distribution function of the test statistic θ_{i,T_i}. Since the tests considered here are one-sided, $p_i = F_{T_i}(\theta_{i,T_i})$ if the test rejects for small values of θ_{i,T_i} and $p_i = 1 - F_{T_i}(\theta_{i,T_i})$ if the test rejects for large values of θ_{i,T_i}. We only consider time series tests with the null of a unit root. We test the following null:

$$H_0: \text{ The time series } i \text{ is unit-root nonstationary} \qquad (i \in \mathbb{N}_N), \tag{1}$$

against the alternative

H_1: For at least one i, the time series is stationary.

($(i \in \mathbb{N}_N)$ is shorthand for $i = 1, \ldots, N$.) The N p-values of the individual time series tests, p_i $(i \in \mathbb{N}_N)$, are combined as follows to obtain a test statistic for panel (non-) stationarity:

$$P_{\chi^2} = -2 \sum_{i=1}^{N} \ln p_i \tag{2}$$

The P_{χ^2} test conveniently imposes minimal homogeneity restrictions on the panel. For instance, the panel can be unbalanced. In

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keeping with most early applications in the literature, we assume independence across i.

Assumption 1 (Cross-sectional independence). For $i \neq j$, $i, j = 1, \ldots, N$, the p-value p_i is independent of p_j.

The following lemma recalls the asymptotic distribution of the test.

Lemma 1. *(Distribution of the P_{χ^2} test).*

Under the null of panel nonstationarity, Assumption 1 and assuming continuous distribution functions of the θ_{i,T_i}, the P_{χ^2} test is, as $\min_i T_i \to \infty$ $(i \in \mathbb{N}_N)$, asymptotically distributed as

$$P_{\chi^2} \to_d \chi^2_{2N}$$

Proof. The proof is an application of the transformation theorem for absolutely continuous random variables (r.v.s). Under H_0 and as $\min_i T_i \to \infty$ $(i \in \mathbb{N}_N)$, $p_i \sim \mathcal{U}[0, 1]$. Let $y = g(p_i) := -2 \ln p_i$. Then, $p_i = g^{-1}(y) = e^{-\frac{1}{2}y}$ and

$$f_{-2\ln p_i}(y) = f_{p_i}(g^{-1}(y))\left|g^{-1'}(y)\right|.$$

Hence, $|g^{-1'}(y)| = \frac{1}{2}e^{-\frac{1}{2}y}$. We have $f_{p_i}(g^{-1}(y)) = 1 \ \forall \ g^{-1}(y) \in [0, 1]$. This implies $f_{-2\ln p_i}(y) = \frac{1}{2}e^{-\frac{1}{2}y}$. The density of a χ^2_2 r.v. is $f_{\chi^2_2}(y) = \frac{1}{2\Gamma(1)}e^{-\frac{y}{2}}$. Recall that $\Gamma(1) = 1$. So, $f_{\chi^2_2}(y) = \frac{1}{2}e^{-\frac{y}{2}}$.

We have shown that $f_{-2\ln p_i}(y) = f_{\chi^2_2}(y)$. The proof is complete because the sum of N *independent* χ^2_R r.v.s is distributed as χ^2_{NR}. □

The maintained assumption of cross-sectional independence is, of course, restrictive in most applications. However, we anticipate that the size distortions to be derived below would, if anything, be higher if cross-sectional dependence were additionally present (see, e.g., Hlouskova and Wagner, 2006). Our results should therefore be viewed as conservative. We further see from the proof that the test has a well-defined asymptotic distribution (for $\min_i T_i \to \infty$) for finite N. This is attractive because in many applications, the assumption that $N \to \infty$ may not be a natural one.

3. The Error-in-Rejection Probability of the combination test

As should be clear from the previous discussion, any unit root test for which p-values are available can be used to compute the P_{χ^2} test statistic. The Dickey and Fuller (1979) test is a popular choice. It is well-known that the (first-order) asymptotic approximation F to the exact finite T_i null distribution of the test statistics, F_{T_i}, need not be accurate. This is because the null hypothesis (1) is not a simple one (and the available test statistics are not pivotal). H_0 is satisfied by all unit root nonstationary processes

$$y_{i,t} = y_{i,t-1} + u_{i,t} \qquad (i \in \mathbb{N}_N)$$

where the errors $u_{i,t}$ can be from a wide class of dependent and heterogeneous sequences. See, for instance, the fairly general mixing conditions on $u_{i,t}$ of Phillips (1987). Hence, the p-values of the test need no longer be uniformly distributed on the unit interval, even if the true Data Generating Process (DGP) of the time series is from the null hypothesis set of unit root nonstationary processes. Thus, the assumptions required for validity of Lemma 1 need no longer be met.

As we argue in this section, this fact can explain the counterintuitive finding of a deteriorating performance of the P_{χ^2} test with increasing N. Table 1 summarizes selected Monte Carlo results on the ERP of the P_{χ^2} test reported in the literature. All find $\text{ERP}_N(\alpha) = |R(\alpha, N) - \alpha|$ to increase with N.

Table 1
Simulated Type I error rates for the P_{χ^2} test

N	5	10	25	30	50	60	100
Maddala and Wu (1999)			.044		.107		.131
Choi (2001)	.050	.070	.090		.090		.130
Hanck (2007)		.035	.031		.021		.014
Hlouskova and Wagner (2006)		.090	.110		.120		.145
Choi (2006)				.051		.042	.037

Note: All results are for the nominal 5% level.

We propose the following modeling assumption to investigate this behavior.

Assumption 2 (Generalized *p*-value distribution). For finite T_i, the p-values are distributed as $\tilde{p}_i \sim \mathcal{U}[a, b]$, where $a \geq 0$, $b \leq 1$ and $a < b$, $(i \in \mathbb{N}_N)$.

Since the exact distribution of the test statistics is generally unknown, so is the exact p-values' distribution. The assumption is, however, convenient for modeling purposes. First, letting $a \to 0$ and $b \to 1$, it comprises the asymptotic result as a limiting case. Second, it is easy to characterize the ERP of a single time series test in terms of a and b. More precisely, since a rejection at level α is equivalent to a p-value $p < \alpha$,

$$\mathrm{P}(F_{\tilde{p}_i} < \alpha) = R(\alpha, 1) = \begin{cases} 0 & \text{for} \quad a > \alpha \\ \dfrac{\alpha - a}{b - a} & \text{for} \quad a < \alpha \text{ and } b > \alpha \\ 1 & \text{for} \quad b < \alpha \end{cases}$$

In particular, it is possible to model "oversized" unit root tests by taking $\tilde{p}_i \sim \mathcal{U}[0, b]$, where $b < 1$. Intuitively, we remove the p-values corresponding to the test statistics speaking most strongly in favor of H_0. Conversely, $\tilde{p}_i \sim \mathcal{U}[a, 1]$, $a > 0$ represents an "undersized" test. The following lemma derives the density function of $-2\ln\tilde{p}_i$.

Lemma 2. *(Distribution of $-2\ln\tilde{p}_i$).*

Under $\tilde{p}_i \sim \mathcal{U}[a, b]$, the density of $-2\ln\tilde{p}_i$ is given by

$$f_{-2\ln\tilde{p}_i}(y) = \begin{cases} 0 & \text{for} \quad y \in (-\infty, -2\ln b) \\ \dfrac{1}{2(b-a)}e^{-\frac{y}{2}} & \text{for} \quad y \in [-2\ln b, -2\ln a] \\ 0 & \text{for} \quad y \in (-2\ln a, \infty), \end{cases}$$

taking $-\ln a = \infty$ for $a = 0$.

Proof. Again, we can apply the r.v. transformation theorem. Using the notation from Lemma 1, we still have $\tilde{p}_i = g^{-1}(y) = e^{-\frac{1}{2}y}$ and hence $|g^{-1'}(y)| = \frac{1}{2}e^{-\frac{y}{2}}$. $f_{\tilde{p}_i}$ follows immediately from Assumption 2 as $f_{\tilde{p}_i}(g^{-1}(y)) = \frac{1}{b-a}$ for $g^{-1}(y) \in [a, b]$ and 0 otherwise. The support of the r.v. $-2\ln\tilde{p}_i$ follows from solving g^{-1} for the lower and upper bounds of $\tilde{p}_i$. One verifies directly that $f_{-2\ln\tilde{p}_i}(y)$ satisfies $\int_{\mathbb{R}} f_{-2\ln\tilde{p}_i}(\tilde{y})\mathrm{d}\tilde{y} = 1$. □

$f_{-2\ln\tilde{p}_i}(y)$ contains the density of the χ^2_2 distribution as a special case with $a = 0$ and $b = 1$.

We now study the ERP of the P_{χ^2} test for the case $N = 1$, denoted $\text{ERP}_1(\alpha)$. Let c_{α_2} be the critical value of the χ^2_2-distribution at nominal level α, i.e. $\int_0^{c_{\alpha_2}} \frac{1}{2}e^{-\frac{1}{2}\tilde{y}}\,\mathrm{d}\tilde{y} = 1 - \alpha \Rightarrow c_{\alpha_2} = -2\ln\alpha$. Then,

$$R(\alpha, 1) = 1 - \int_{-\infty}^{-2\ln\alpha} f_{-2\ln\tilde{p}_i}(\tilde{y})\mathrm{d}\tilde{y} = \frac{\alpha}{b}$$

As an example, consider the "oversized" case, $\tilde{p}_i \sim \mathcal{U}[0, 0.9]$, and $\alpha = 0.05$. Then, $\text{ERP}_1(0.05) = \left|\frac{0.05(1-0.9)}{0.9}\right| \approx 0.005$, yielding an ERP which would be considered small in most Monte Carlo analyses.