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Estimating economies of scope using the profit function: A dual approach for the normalized quadratic profit function

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ABSTRACT

Theoretical relationships between parameters of the normalized quadratic profit and cost functions are derived allowing for economies of scope to be calculated using profit function estimates. An empirical example confirms that the cost function is recovered using the estimated profit function.

1. Introduction

Economies of scope (EOS) measure the percentage of cost savings of producing several products in a single firm compared to producing the same products separately. The sources of economies of scope lie in the complimentary property among inputs. Since Baumol et al. (1982), economies of scope have become an important concept for measuring cost savings for multiproduct firms. The common approach involves estimating a cost function, and comparing the cost of producing multi-products jointly with the cost of producing all the products individually.

The normalized quadratic functional form is often used in the study of economies of scope (Featherstone and Moss, 1994; Fernandez-Cornejo et al., 1992; Jin et al., 2005; and Cohn et al., 1989). One of the disadvantages of the parametric approach is that the data used to estimate cost functions are not always on the efficient frontier. Because scope economies are defined only on the efficiency frontier, testing economies of scope using data off the frontier could confound scope economies with X-efficiencies (Berger et al., 1993b). In addition, imposing curvature in a profit function is easier than in a cost function. Normally, concavity in outputs and convexity in inputs are imposed for two sub-matrices of the Hessian matrix, and off diagonal sub-matrices are not considered. Using the profit function makes it easier to impose curvature on the off diagonal sub-matrices (Marsh and Featherstone, 2004). Berger et al. (1993a) also argue that measuring scope economies from a cost function doesn't consider whether the output bundle is optimal. Therefore, they suggest that more research should concentrate on estimating economies of scope from the profit function, which includes both the revenue and cost sides of production. Berger et al. (1993a) provided a new concept of optimal scope economies, which determines “whether a firm facing a given set of prices and other exogenous factors should optimally produce the entire array of products or specialize in some of them”. Using an unrestricted profit function, the optimal quantities of outputs can be derived using Hotelling's lemma. If the optimal quantities of outputs are determined to be positive at given exogenous prices, optimal scope economies exist at that point.

Following Berger et al.'s suggestion, we provide a way to estimate economies of scope using the profit function. Different from Berger et al.'s (1993a) approach, we use the classic concept of scope economies that was first provided by Baumol et al. (1982).

2. Duality and recovering cost function from unrestricted profit function

To determine economies of scope (EOS), the cost of producing multiproducts jointly and the sum of the cost to produce these products individually are compared. Economics of scope measure the

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savings that occur if the products are produced jointly rather than separately. Specifically, EOS is:

$$\mathrm{EOS} = \frac{\sum_i C(Y_i) - C(Y)}{C(Y)},$$

where $C(Y_i)$ is the cost of producing only Y_i in a separate firm, and $C(Y)$ is the cost of producing all outputs by a single multiproduct firm. If EOS is positive, economies of scope exist and firms are more cost efficient by diversifying production.

Duality theory indicates that a profit maximizing firm also minimizes cost, and the unrestricted profit function contains the same economic information as the indirect cost function (Mas-Colell et al., 1995). Theoretically, it's possible to link the parameters of the profit function to the parameters in the cost function. Lau (1976) provides Hessian identities where under perfect competition, a restricted profit (cost) function or production function can be recovered from an unrestricted profit function and vice versa. Lusk et al. (2002) examined the relationship between the parameters of production function, unrestricted profit function and restricted profit function empirically. The Hessian identities provide a relationship to determine the quadratic effects but do not provide a mechanism for determining the intercept and the linear terms that are needed to estimate economies of scope from the profit function. We use the normalized quadratic functional form to determine that relationship. Starting with a cost function, we use the maximization process to calculate the unrestricted profit function. If the parameters of profit function can be expressed using the parameters of the cost function, an inverse relationship can be obtained, which expresses the parameters of the cost function using the parameters from the profit function.

3. Theoretical relationship between cost and unrestricted profit functions

Suppose that we have a normalized quadratic indirect cost function $C(W,Y)$ that is continuous in (W,Y) and differentiable in W and Y, linear homogenous and concave in W, and convex in Y. The normalized cost function with $n+1$ inputs and m outputs is expressed as:

$$C(W,Y) = b_0 + \underset{1*n}{B} * \underset{n*1}{W} + \underset{1*m}{A} * \underset{m*1}{Y} + 0.5*\underset{1*n}{W'} * \underset{n*n}{BB} * \underset{n*1}{W} + 0.5*\underset{1*m}{Y'} * \underset{m*m}{CC} * \underset{m*1}{Y} + \underset{1*n}{W'} * \underset{n*m}{AA} * \underset{m*1}{Y}, \qquad (1)$$

where $C(W,Y)$ is the normalized cost, W is a vector of input normalized prices and Y is the measure of output. The cost and input prices are normalized by the $n+1$ input price which imposes the homogeneity condition. Formally,

$$B = [b_1 b_2 \ldots\ldots b_n]$$

$$W = [w_1 w_2 \ldots\ldots w_n]$$

$$A = [a_1 a_2 \ldots\ldots a_m]$$

$$Y = [y_1 y_2 \ldots\ldots y_n]$$

$$\mathrm{BB} = \begin{bmatrix} b_{11} & b_{12} & \ldots\ldots & b_{1n} \\ b_{21} & b_{22} & \ldots\ldots & b_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ b_{n1} & b_{n2} & \ldots\ldots & b_{nn} \end{bmatrix} \text{ where } b_{ij} = b_{ji} \text{ to satisfy symmetry}$$

$$\mathrm{CC} = \begin{bmatrix} c_{11} & c_{12} & \ldots\ldots & c_{1m} \\ c_{21} & c_{22} & \ldots\ldots & c_{2m} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{m1} & c_{n2} & \ldots\ldots & c_{mm} \end{bmatrix} \text{ where } c_{ij} = c_{ji} \text{ to satisfy symmetry, and}$$

$$\mathrm{AA} = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1m} \\ a_{21} & a_{22} & \ldots\ldots & a_{2m} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \ldots\ldots & a_{nm} \end{bmatrix}.$$

The Hessian matrix of the normalized quadratic cost function for input prices and output quantities are BB and CC respectively. The curvature and symmetry conditions together imply that BB and CC are negative semi-definite symmetric matrices and positive semi-definite symmetric matrices, respectively.

Assume both input and output markets are perfectly competitive, the unrestricted profit function can be obtained as a result of following maximization problem:

$$\Pi = \max P*Y - C(W,Y). \qquad (2)$$

where P is a vector of exogenous output prices, $P=[p_1\, p_2 \ldots\ldots p_m]$. The first order conditions allow us to determine the optimal output $\frac{\partial \Pi}{\partial Y} = P' - \frac{\partial C(W,Y)}{\partial Y} = 0$ by solving a set of equations. For the normalized quadratic cost function (1), the first order conditions are:

$$P' = A' + \mathrm{CC}*Y + \mathrm{AA}'*W, \qquad (3)$$

and the optimal output quantities are determined by solving for Y are:

$$Y* = \mathrm{CC}^{-1}*(P' - A' - \mathrm{AA}'*W). \qquad (4)$$

Plugging $Y*$ into the original cost function (1) to solve for the cost at the optimal output quantities:

$$\begin{aligned} C(W,Y*) = {} & b_0 + \underset{1*n}{B} * \underset{n*1}{W} + \underset{1*m}{A} * \underset{m*1}{\mathrm{CC}^{-1}*(P' - A' - \mathrm{AA}'*W)} + 0.5*\underset{1*n}{W'} * \underset{n*n}{\mathrm{BB}} * \underset{n*1}{W} \\ & + 0.5*\underset{1*m}{\left[\mathrm{CC}^{-1}*(P' - A' - \mathrm{AA}'*W)\right]'} * \underset{m*m}{\mathrm{CC}} * \underset{m*1}{\left[\mathrm{CC}^{-1}*(P' - A' - \mathrm{AA}'*W)\right]} \\ & + \underset{1*n}{W'} * \underset{n*m}{\mathrm{AA}} * \underset{m*1}{\mathrm{CC}^{-1}*(P' - A' - \mathrm{AA}'*W)}. \end{aligned} \qquad (5)$$

Expanding via multiplication results in:

$$\begin{aligned} & C(W,Y*) \\ & = b_0 + B*W + \left(A*\mathrm{CC}^{-1}*P' - A*\mathrm{CC}^{-1}*A' - A*\mathrm{CC}^{-1}*\mathrm{AA}'*W\right) \\ & + 0.5*W'*\mathrm{BB}*W + 0.5*\left[(P' - A' - \mathrm{AA}'*W)'*(\mathrm{CC}^{-1})'\right]*(P' - A' - \mathrm{AA}'*W) \\ & + \left(W'*\mathrm{AA}*\mathrm{CC}^{-1}*P' - W'*\mathrm{AA}*\mathrm{CC}^{-1}*A' - W'*\mathrm{AA}*\mathrm{CC}^{-1}*\mathrm{AA}'*W\right). \end{aligned} \qquad (6)$$

Because CC and BB are symmetric matrices, $(\mathrm{CC}^{-1})'$ is equal to CC^{-1} and $(\mathrm{BB})'$ is equal to BB. Further expanding Eq. (6), the cost function is:

$$\begin{aligned} C(W,Y*) = {} & b_0 - A*\mathrm{CC}^{-1}*A' + 0.5*A*\mathrm{CC}^{-1}*A' \\ & + B*W - A*\mathrm{CC}^{-1}*\mathrm{AA}'*W + 0.5*W'*\mathrm{AA}*\mathrm{CC}^{-1}*A' \\ & + 0.5*A*\mathrm{CC}^{-1}*\mathrm{AA}'*W - W'*\mathrm{AA}*\mathrm{CC}^{-1}*A' + A*\mathrm{CC}^{-1}*P' \\ & - 0.5*A*\mathrm{CC}^{-1}*P' - 0.5*P*\mathrm{CC}^{-1}*A' + 0.5*W'*\mathrm{BB}*W \\ & + 0.5*W'*\mathrm{AA}*\mathrm{CC}^{-1}*\mathrm{AA}'*W - W'*\mathrm{AA}*\mathrm{CC}^{-1}*\mathrm{AA}'*W \\ & + 0.5*P*\mathrm{CC}^{-1}*P' - 0.5*W'*\mathrm{AA}*\mathrm{CC}^{-1}*P' \\ & - 0.5*P*CC^{-1}*\mathrm{AA}'*W + W'*\mathrm{AA}*\mathrm{CC}^{-1}*P'. \end{aligned} \qquad (7)$$

Each term in Eq. (7) is a scalar, thus we can simplify the above equation to:

$$\begin{aligned} C(W,Y*) = {} & b_0 - 0.5*A*\mathrm{CC}^{-1}*A' + \left(B - A*\mathrm{CC}^{-1}*\mathrm{AA}'\right)*W \\ & + 0.5*W'*\left(\mathrm{BB} - \mathrm{AA}*\mathrm{CC}^{-1}*\mathrm{AA}'\right)*W \\ & + 0.5*P*\mathrm{CC}^{-1}*P'. \end{aligned} \qquad (8)$$