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Stochastic dominance representation of optimistic belief: Theory and applications ☆

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ABSTRACT

This note gives a stochastic dominance representation of more optimistic belief in rank dependent expected utility. Applying this observation, we can demonstrate the effect of more optimistic belief on economic decisions and equilibria under risk making comparative static analysis.

1. Introduction

Optimism and pessimism play a crucial role in decision-making under risk, but expected utility theory allows no role for optimism or pessimism. Rank dependent expected utility theory (RDEU), introduced by Quiggin (1981, 1982), is one of the most widely used alternatives to expected utility theory, and allows for direct representation of these risk attitudes. This note uses comparative static analysis, based on a stochastic dominance representation of optimism and pessimism, to describe the implications of optimism and pessimism for economic decisions and equilibria under risk.

Because RDEU maintains a separation between probability weights and utility over outcomes, as in expected utility, it is possible to interpret optimism and pessimism as properties of beliefs, with the utility function capturing preferences over outcomes. Accordingly, we will refer to optimistic and pessimistic beliefs. However, the analysis to follow is unaffected if probabilities and probability weights are treated as a component of preferences.

In this note, the optimism of beliefs is ranked by monotone likelihood ratio dominance between corresponding decision weights. It is known that this is useful to obtain clear comparative static predictions in expected utility theory. As in Quiggin (1991), rank dependent expected utility can be regarded as “expected utility with respect to a transformed probability distribution”. This observation suggests that we can also obtain them in RDEU. Also, monotone likelihood ratio dominance is defined as log-supermodularity between compared probability density functions. As Athey (2002) demonstrated, it plays an important role in comparative static analysis under risk. These results suggest that it will be useful to consider the effect of optimistic belief on economic decisions and equilibria under risk. We confirm this expectation applying a risk-neutral characterization, which is a natural extension of the analysis in expected utility such as Ohnishi and Osaki (2006a) and Osaki (2005).[1]

Our main concern is how shifts in probability weighting functions representing more optimistic belief in rank dependent expected utility theory influence economic decisions and equilibria under risk.[2] By contrast, most previous studies in this area have focused on whether comparative static results in expected utility theory are preserved under generalized expected utility theory. Quiggin (1991, 1995) determined conditions and restrictions for the preservation of comparative static results. Schlee (1994) claimed that monotone likelihood ratio dominance may not have clear comparative static predictions under

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[1] Ohnishi and Osaki (2006b) and Quiggin and Chambers (2007) demonstrated comparative static analysis in non-expected utility theory applying the property of (log-)supermodularity.

[2] It is clear that we can demonstrate the effect of more pessimistic belief on economic decisions and equilibria by the definition of optimism and pessimism in this note. Thus we basically mention the case of optimistic belief in the following.

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generalized expected utility theory. Work closer to our approach is that of Bleichrodt and Eeckhoudt (2005), who demonstrated that optimistic belief in rank dependent expected utility theory influence precautionary saving and exhibited the effect of higher-order derivatives of utility functions on it. Introducing our representation of optimistic belief, we can predict the effect of optimistic belief on many economic problems under risk by comparative static predictions.

The organization of our note is as follows. Section 2 briefly gives some preliminaries and provides a stochastic dominance representation of optimistic belief. In Section 3, we introduce a risk-neutral characterization and obtain two comparative static predictions applying this property. Section 4 is a concluding comment.

2. Theory

First, we give a representation of rank dependent expected utility (RDEU), a preference relation defined over discrete random variables. A discrete random variable is an outcome vector $\boldsymbol{x}=(x_1, x_2,\dots x_S)$ with a corresponding probability vector $\boldsymbol{p}=(p_1, p_2,\dots, p_S)$. Every probability is strictly positive, $p_s>0$, and sum to one, $\sum_{s=1}^{S} p_s=1$. Without loss of any generality, we assume that the outcomes are ranked in ascending order, $x_1<x_2<\dots<x_S$. A Decision Maker (DM) has RDEU representation if the discrete random variable $(p_1, x_1; p_2, x_2;\dots; p_S, x_S)$ is evaluated by

$$V(p_1,x_1;p_2,x_2;\dots;p_S,x_S) := \sum_{s=1}^{S} d_s^q u(x_s), \tag{1}$$

where $\boldsymbol{d}^q=(d_1^q, d_2^q,\dots, d_S^q)$ is a decision weight vector with respect to a probability weighting function (PWF) q, and u is a utility function. The decision weight d_s^q is

$$d_s^q = q(p_1+p_2+\dots+p_s)-q(p_1+p_2+\dots+p_{s-1}), \tag{2}$$

and $d_1^q=q(p_1)$. For notational ease, the cumulative probability is denoted as the capital letter, $P_n=\sum_{s=1}^{n} p_s$. The probability weighting function q is increasing in P with $q(0)=0$ and $q(1)=1$. Expected utility (EU) representation corresponds to the linear PWF $q(P)=P$. The utility function is increasing in x. We note that concavity of the utility function is not necessary for our analysis.

We define a "more optimistic" belief as follows:

Definition 2.1. A probability weighting function q is more optimistic than r if there exists an increasing and convex function ϕ such that $q=\phi\bigcirc r$.

We give a couple of explanations on more optimistic belief. First, more optimistic beliefs underweight the cumulative probability of worse outcomes, $q(P)\le r(P)$ for all P. Second, more pessimistic belief can represent increasing and concave transformations. Thus our analysis can be also applied to the case of more pessimistic belief relations. Finally, as shown by Chew et al. (1987), concave PWFs which is more pessimistic than original distributions, characterize the strong risk aversion, aversion to mean-preserving increase in risk (Rothschild and Stiglitz, 1970). In a recent paper, Ryan (2006) obtained it under weaker conditions and also displayed a nice summary of risk aversion in RDEU.

As will be shown later, more optimistic belief can be characterized by the following stochastic dominance:

Definition 2.2. Let us consider two probability vectors $\boldsymbol{p}^1=(p_1^1, p_2^1,\dots, p_S^1)$ and $\boldsymbol{p}^2=(p_1^2, p_2^2,\dots, p_S^2)$. The probability vector $\boldsymbol{p}^2$ dominates $\boldsymbol{p}^1$ in the sense of monotone likelihood ratio dominance, if

$$\frac{p_t^2}{p_s^2}\ge\frac{p_t^1}{p_s^1} \tag{3}$$

for all $s<t$.

Monotone likelihood ratio dominance (MLRD) is a stronger stochastic dominance than First-order Stochastic Dominance (FSD) (Shaked and Shanthikumar (1994), Müller and Stoyan (2002)).

Theorem 2.1. *A probability weighting function q is more optimistic than r, if and only if the corresponding decision weight d^q with respect to the probability weighting function q dominates d^r in the sense of monotone likelihood ratio dominance.*

Proof. We define $R_s:=r(P_s)$. By noting that $R_s\le R_t$ for all $s<t$ because of r increasing in P, convexity of ϕ is equivalent to

$$\frac{\phi(R_t)-\phi(R_{t-1})}{R_t-R_{t-1}}\ge\frac{\phi(R_s)-\phi(R_{s-1})}{R_s-R_{s-1}}. \tag{4}$$

Since $q=\phi\bigcirc r$, we have

$$\frac{q(P_t)-q(P_{t-1})}{q(P_s)-q(P_{s-1})}\ge\frac{r(P_t)-r(P_{t-1})}{r(P_s)-r(P_{s-1})}. \tag{5}$$

This can be rewritten $d_t^q/d_s^q\ge d_t^r/d_s^r$ by the definition of the decision weight. □

Quiggin (1995) pointed out that the probability weighting function derived from a convex transformation dominates the original distribution function in the sense of MLRD. This result is included as a corollary of the theorem. The theorem suggests the comparative static results of MLRD changes in EU applicable to those of the shift to a more optimistic belief in RDEU. This means that any of the comparative static results of Athey (2002) and others for MLRD changes in EU have an interpretation in RDEU. Actually, we demonstrate two comparative static applications using a risk-neutral characterization in the following section.

3. Applications

In this section, we give a risk-neutral characterization of optimistic belief, and then demonstrate two comparative static predictions applying this property. First, we show that more optimistic representative investors lead to increases in equilibrium asset prices. Second, we display that more optimistic decision makers behave in a more risk-tolerant manner under comonotonic background risk when utility functions exhibit decreasing absolute risk aversion.

3.1. A risk-neutral characterization

We define the risk-neutral decision weight with respect to q as follows:

$$\hat{d}_s^q := \frac{d_s^q u'(x_s)}{\sum_{s=1}^{S} d_s^q u'(x_s)}. \tag{6}$$

It is clear that the risk-neutral decision weight is also the probability of outcome x_s, since it is strictly positive, $\hat{d}_s^q>0$ and sum to one, $\sum_{s=1}^{S}\hat{d}_s^q=1$. By the definition, we can easily obtain the following equivalence:

$$\frac{d_t^q}{d_s^q}\ge\frac{d_t^r}{d_s^r}\Leftrightarrow\frac{\hat{d}_t^q}{\hat{d}_s^q}=\frac{d_t^q u'(x_t)}{d_s^q u'(x_s)}\ge\frac{d_t^r u'(x_t)}{d_s^r u'(x_s)}=\frac{\hat{d}_t^r}{\hat{d}_s^r}. \tag{7}$$

We give a formal statement of the above:

Property 3.1. *A probability weighting function q is more optimistic than r, if and only if the corresponding risk-neutral decision weight vector $\hat{\mathbf{d}}^q$ with respect to the probability weighting function q dominates $\hat{\mathbf{d}}^r$ in the sense of monotone likelihood ratio dominance.*

This property suggests that the MLRD order between probability distributions is preserved for the corresponding risk-neutral decision weights. Risk-neutral valuation is a central idea of modern finance. Many valuation formulas appeared in economics and finance, can be given by the risk-neutral representation. Thus, the property is useful for investigating the effect of optimistic beliefs. The stochastic dominance representation of optimistic belief indicates that any