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Overnight interest rates and aggregate market expectations

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Abstract

This paper introduces an entropy approach to measuring market expectations with respect to overnight interest rates in an inter-bank money market. The findings for the Turkish 2000–2001 borrowing crisis suggest that a dynamic, non-extensive entropy framework provides a valuable insight into the degree of aggregate market concerns during the crisis.

Keywords: Non-additive entropy; Tsallis entropy; q-Gaussian distribution

JEL classification: G0; G1; C40

1. Introduction

The Turkish financial crisis that peaked in February 2001 is an example of extreme risks in financial markets. The events that led to the crisis began in 1999 with a government reform program that was supposed to curb inflation by fixing the value of the Turkish Lira against a basket of foreign currencies. The structure of the program was such that it left the interest rate to be market determined. In other words, the reform implied that the volatility of interest rates would increase over the course of the program implementation (Gençay and Selçuk, 2006). However, the increase in the (simple annual) overnight interest rate that followed was unprecedented and reached 873% on December 1, 2000, 2058% on February 20, 2001, and 4019% on February 21, 2001. The government immediately abandoned the *tablita* program, the Turkish Lira depreciated against the U.S. dollar by 40% in one week, and the interest rate declined to stable levels.

We study the dynamics of market expectations of the overnight interest rate during the crisis using a non-extensive (or Tsallis) entropic measure.[1] We argue that the sentiment of a financial market can be summarized through the aggregation of the subjective expectations of its participants. If the expectations of market participants are highly dispersed and independent, extreme interest rate movements are less likely to occur. If however, market participants have highly dependent and less dispersed expectations, the aggregate market sentiment could drive the interest rate to extraordinary levels. Our approach extracts aggregate market expectations from a past sequence of interest rates via time-dependent Tsallis entropy. By utilizing this particular measure we concentrate on long-range, time-dependent, interactive instability in the market (Gell-Mann and Tsallis, 2004; Martin et al., 2000). Moreover, the Tsallis (or a q-Gaussian) distribution that is estimated by maximizing the Tsallis entropy can capture the frequency of extreme events together with ordinary frequencies satisfactorily (Borland, 2002; Gell-Mann and Tsallis, 2004). The findings show that from an entropic perspective market concerns between December 2000 and February 2001 were particularly strong

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[1] Gell-Mann and Tsallis (2004) provide an excellent survey of this methodology and its interdisciplinary applications. In some other applications, Stutzer (2000) and Stutzer and Kitamura (2002) study option and asset pricing through an entropic methodology.

Table 1
Distribution of aggregate market expectations—overnight interest rates

Date	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	TE
Feb 2, 2000	0	0.04	0.03	0.01	0.01	0.01	0.05	0.44	0.17	0.24	0.513
Mar 2, 2000	0	0.06	0.03	0.02	0.05	0.02	0.06	0.45	0.17	0.14	0.521
Apr 3, 2000	0	0.07	0.08	0.05	0.06	0.02	0.06	0.45	0.17	0.04	0.521
May 2, 2000	0.01	0.08	0.12	0.09	0.07	0.02	0.05	0.44	0.12	0	0.528
Jun 2, 2000	0.01	0.08	0.16	0.12	0.10	0.02	0.05	0.42	0.04	0	0.535
Jul 3, 2000	0.01	0.09	0.21	0.13	0.11	0.04	0.03	0.38	0	0	0.543
Aug 2, 2000	0.02	0.08	0.20	0.19	0.13	0.05	0.04	0.03	0.26	0	0.566
Sep 1, 2000	0.02	0.16	0.26	0.21	0.12	0.05	0.02	0.16	0	0	0.565
Oct 2, 2000	0.02	0.17	0.29	0.24	0.13	0.06	0.02	0.06	0.01	0	0.559
Nov 2, 2000	0.02	0.16	0.32	0.26	0.13	0.06	0.03	0.02	0.01	0	0.550
Nov 29, 2000	0.20	0.59	0.13	0.06	0.01	0.01	0	0	0	0	0.446
Nov 30, 2000	0.70	0.25	0.03	0.01	0.01	0	0	0	0	0	0.354
Dec 1, 2000	0.97	0.02	0	0.01	0	0	0	0	0	0	0.046
Dec 26, 00–Feb 19, 01	0.92	0.07	0	0.01	0	0	0	0	0	0	0.126
Feb 20, 2001	0.98	0.01	0	0.01	0	0	0	0	0	0	0.039
Feb 21, 2001	0.98	0.01	0.01	0	0	0	0	0	0	0	0.031

The time-dependent Tsallis entropy (TE) is calculated with a moving window of 200 days for the overnight interest rate. $s_1,\ldots, s_{10}$ denote non-overlapping intervals (states). The lower boundary of s_1 is the minimum of the moving window. Accordingly, the upper boundary of s_{10} is the maximum of the moving window. Aggregate expectation probabilities (p_i) are calculated from the ratio between the number of observations in each interval and the total number of observations in the moving window. The maximum entropy (expectations heterogeneity) corresponds to equal probability of 10% for each state. The minimum entropy (expectations homogeneity) occurs when all observations concentrate in one particular state such that one state receives 100% of the probability. In this particular case above, distribution of expectations is more evenly distributed until November 29, 2000 which becomes more concentrated towards December 1, 2000 in states $s_1,\ldots, s_4$. The increased concentration leads to a reduction in the entropy. The entropy is bounded between [0,1.48] for $q=2.66$.

and that subsequent high interest rate levels were to some extent predictable.

2. *q*-Gaussianity and Tsallis entropy

We will motivate our approach of modeling aggregate market expectations through a time-dependent Tsallis entropy by relating how the number of states (or regimes) in a market translate themselves into a probability distribution of the aggregate market sentiment. One well-known entropy is the Shannon information measure (S_S):

$$\begin{aligned} S_S(f(x)) &= \int f(x) \quad \ln \left[\frac{1}{f(x)}\right] \mathrm{d}x \\ &= -\int f(x) \quad \ln \quad [f(x)] \mathrm{d}x, \end{aligned} \tag{1}$$

or, in discrete setting S_S is,

$$S_S = -\sum_{i=1}^{n} p_i \ \ln \ p_i, \quad \sum_{i=1}^{n} p_i = 1 \tag{2}$$

where the number of states $i=1,\ldots, n$, p_i is the probability of outcome i, and n is the number of states. Namely, the entropy is the sum over the product of the probability of outcome (p_i) times the logarithm of the inverse of p_i. This is also called i's surprisal and the entropy of x is the expected value of its outcome's surprisal. It is worthwhile to note that if two states A and B are independent from one another, $p(A\cup B)=p(A)p(B)$, then S_S is additive S_S $(A\cup B)=S_S$ $(A)+S_S$ (B).

Tsallis (1988) entropy (S_q) is a generalization to a non-additive measure

$$S_q(f(x)) = \frac{1-\int f(x)^q dx}{q-1} \tag{3}$$

where q is a measure of non-additivity such that S_q $(A\cup B)=S_q$ $(A)+S_q$ $(B)-(1-q)S_q(A)S_q$ (B). Tsallis entropy recovers the Shannon entropy when $q\rightarrow 1$ such that $\lim_{q\rightarrow 1} S_q=S_S$.

S_q in its discrete version can be written as

$$S_q = \frac{1-\sum_{i=1}^{n} P_i^a}{q-1}. \tag{4}$$

The maximum entropy principle for Tsallis entropy under the constraints

$$\int f(x)\mathrm{d}x = 1, \quad \frac{\int x^2 f(x)^q \mathrm{d}x}{\int f(y)^q \mathrm{d}y} = \sigma^2 \tag{5}$$

yields[2] the q-Gaussian probability density function

$$f(x) = \frac{\exp_q\left(-\beta_q x^2\right)}{\int \exp_q\left(-\beta_q x^2\right)\mathrm{d}x} \propto \left[1+(1-q)\left(-\beta_q x^2\right)\right]^{\frac{1}{1-q}} \tag{6}$$

where β_q is a function of q and $\exp_q$ (x) is the q-exponential function defined by

$$\exp_q(x) = \begin{cases} [1+(1-q)x]^{\frac{1}{1-q}} & \text{if } 1+(1-q)x>0 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

[2] See, for instance, Suyari (2006).