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Problems related to over-identifying restrictions for structural vector error correction models

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Abstract

If cointegrated variables are involved in a structural VAR analysis, vector error correction models offer a convenient framework for imposing structural long-run and short-run restrictions. Problems related to over-identifying restrictions in these models and possible solutions are discussed.

Keywords: Cointegration; Vector autoregressive process; Vector error correction model; Impulse responses

JEL classification: C32

1. Introduction

In structural vector autoregressive (SVAR) modelling long-run restrictions are often used in addition to short-run restrictions to identify the shocks and impulse responses of interest. In particular, if cointegrated variables are present, the cointegration properties may be useful in specifying the number of shocks with permanent and transitory effects. Vector error correction models (VECMs) and the framework laid out by King et al. (1991) offer a possible setup for imposing identifying restrictions. In this note I will argue that these restrictions require some care in doing inference for impulse responses. In particular, certain over-identifying restrictions are not possible because they imply a singular residual covariance matrix which is usually ruled out by assumption and is also not plausible from a theoretical point of view. In other words, some over-identifying restrictions may not be possible because they are outside the admissible parameter space. This of course also means that associated t-ratios cannot be interpreted in the usual way. Unfortunately, it is not always obvious which over-identifying restrictions are possible and which ones are not admissible. Therefore I will discuss conditions that can help to see more easily which restrictions are not feasible. It may be worth pointing out that the problem also affects the impulse responses. In particular, the interpretation of confidence intervals around impulse response functions needs some care.

The study is structured as follows. In the next section the model setup for structural modelling with cointegrated VAR processes will be presented. Estimation of the models including the associated problems and possible solutions are discussed in Section 3. Proofs of two propositions are presented in Section 4. The structural VECM framework of the present article was proposed by King et al. (1991) and is also discussed in detail in Lütkepohl (2005, Chapter 9).

In the following the usual $I(d)$ terminology related to integrated and cointegrated processes is used. Moreover, the natural logarithm is abbreviated as log. For a suitable matrix A, rk(A), det(A) and $A_\perp$ denote the rank, the determinant and an orthogonal complement of A, respectively. Moreover, vec is the column stacking operator which stacks the columns of a matrix in a column vector and vech is the column stacking operator for symmetric square matrices which stacks the columns from the main diagonal downwards only. The $(n\times n)$ identity matrix is signified as I_n and $0_{n\times m}$ denotes an $(n\times m)$ zero matrix.

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2. The model setup

It is assumed that all variables are at most $I(1)$ and that the data generation process can be represented as a VECM of the form

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_{p-1}\Delta y_{t-p+1} + u_t,\ t = 1, 2, \ldots, \quad (2.1)$$

where y_t is a K-dimensional vector of observable variables and α and β are $(K\times r)$ matrices of rank r. More precisely, β is the cointegration matrix and r is the cointegrating rank of the process. The Γ_j's, $j=1,\ldots,p-1$, are $(K\times K)$ short-run coefficient matrices and u_t is a white noise error vector with mean zero and nonsingular covariance matrix $\sum_u$, $u_t \sim (0_{K\times 1}, \sum_u)$. Moreover, $y_{-p+1} \ldots, y_0$ are assumed to be fixed initial conditions. Although in practice there will usually also be deterministic terms such as nonzero means or polynomial trends, it will be assumed in the following that such terms are absent. They do not play a role in impulse response analysis which is the focus of this study.

Impulse responses are often used to study the relationships between the variables of a dynamic model such as Eq. (2.1). In this context, identifying structural innovations which induce responses of the variables reflecting the actual ongoings in a system is an important task. In the present VECM framework, the so-called B-model setup is typically used (Lütkepohl, 2005, Chapter 9). It is assumed that the structural innovations, say ε_t, have zero mean and identity covariance matrix, $\varepsilon_t \sim (0_{K\times 1}, I_K)$, and they are linearly related to the u_ts such that $u_t = B\varepsilon_t$ hence, $\sum_u = BB'$.

Using Johansen's version of Granger's representation theorem (Johansen, 1995), it can be shown that the long-run effects of the structural innovations are given by ΞB, where

$$\Xi = \beta_\perp \left[\alpha_\perp' \left(I_K - \sum_{i=1}^{p-1}\Gamma_i\right)\beta_\perp\right]^{-1} \alpha_\perp'$$

is a $(K\times K)$ matrix of rank $K-r$ (e.g., Lütkepohl, 2005, Section 9.2). The structural innovations ε_t have nonsingular covariance matrix and, hence, the matrix B must also be nonsingular. Thus, $rk(\Xi B) = K - r$ and there can be at most r zero columns in the matrix ΞB. In other words, at most r of the structural innovations can have transitory effects only and at least $K-r$ of them must have permanent effects. The transitory shocks may be identified, for example, by placing zero restrictions on B directly and thereby specifying that certain shocks have no instantaneous impact on some of the variables. Typically zero restrictions are imposed on the instantaneous and long-run effects. Precise conditions for local just-identification may be found in Lütkepohl (2005, Proposition 9.4).

3. Estimation

Assuming that the lag order, $p-1$, the cointegrating rank, r, and structural identifying restrictions are given, a VECM can be estimated by a Gaussian maximum likelihood (ML) or quasi ML procedure. Under usual assumptions, the ML estimator of B, $\widehat{B}$ say, is consistent and asymptotically normal,

$$\sqrt{T}\,\mathrm{vec}\left(\widehat{B} - B\right) \xrightarrow{d} \mathcal{N}\left(0, \Sigma_{\widehat{B}}\right). \quad (3.1)$$

Expressions for the covariance matrix of the asymptotic distribution in terms of the model parameters can be obtained by working out the corresponding information matrix (see Vlaar, 2004). For practical purposes, bootstrap methods are in common use for inference in this context.

The result in Eq. (3.1) implies that the t-ratios of elements with regular asymptotic distributions can be used for assessing the significance of individual parameters, provided the corresponding over-identifying zero restriction is a valid one. In other words, the zero value of the corresponding parameter must be within the admissible parameter space. This is not always obvious as I will argue in the following. I will now present a criterion for deciding on inadmissible zero restrictions. A proof is given in Section 4.

Proposition 1. *In the model Eq.* (2.1)*, suppose the* $(K\times(K-r))$ *matrix* $\alpha_\perp$ *is such that all sets of* $K-r$ *rows are nonsingular* $((K-r)\times(K-r))$ *matrices and there are* $r^* \le r$ *transitory shocks. Then the number of admissible zero restrictions placed on a column of B associated with a transitory shock cannot be greater than* $r-1$. □

Thus, in general, there cannot be more than $r-1$ zero restrictions on a column of B associated with a transitory shock. Clearly, this condition is easy to check if the cointegrating rank r is known. For example, if $r=1$ and there is one transitory shock (i.e., $r^*=r$), then there cannot be any zero restriction on the column of B corresponding to the transitory shock. If $r=2$ and there are two transitory shocks, there can be at most one zero restriction on each of the two columns of B corresponding to the transitory shocks. For example, in a three-dimensional system with just-identifying restrictions

$$\Xi B = \begin{bmatrix} * & 0 & 0 \\ * & 0 & 0 \\ * & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} * & * & * \\ * & * & 0 \\ * & * & * \end{bmatrix} \quad (3.2)$$

the last two shocks are transitory. Hence, there can only be at most one zero restriction on each of the last two columns of B. Thus, no further zero restriction can be imposed on the last column because there is already one identifying zero restriction on this column. Hence, t-ratios cannot be used to check the significance of the remaining two elements in the last column of B.

If $r=2$ and there is only one transitory shock ($r^*=1$), say the last one in ε_t, then we may have just-identifying restrictions of the form

$$\Xi B = \begin{bmatrix} * & * & 0 \\ * & * & 0 \\ * & * & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} * & 0 & * \\ 0 & * & * \\ * & * & * \end{bmatrix}.$$