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Real-time rational expectations and indeterminacy

Paul Shea *

335T Gatton Building, University of Kentucky, Lexington, KY 40536, United States

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Abstract

This paper uses the technique of Lubik and Schorfheide [Lubik, T., Schorfheide, F., 2004. Testing for indeterminacy: an application to U.S. monetary policy. The American Economic Review 94 (1), 190–217] to test for indeterminacy in a New Keynesian Model. Using real-time, instead of revised data on the output gap, the results suggest indeterminacy both before 1979 and after 1982.

Keywords: Indeterminacy; Monetary policy rules; Real-time data; Bayesian estimation

JEL classifications: C11; C62; E32; E52

1. Introduction

The log-linearized New Keynesian (hereafter "NK") model consists of the following five equations:[1]

$$\tilde{x}_t = E_t[\tilde{x}_{t+1}] - \tau\left(\tilde{R}_t - E_t[\tilde{\pi}_{t+1}]\right) + g_t \tag{1.1}$$

$$\begin{aligned} \tilde{\pi} &= \beta E_t[\tilde{\pi}_{t+1}] + \kappa(\tilde{x}_t - z_t) \\ g_t &= \rho_g g_{t+1} + \varepsilon_{g,t} \\ z_t &= \rho_z z_{t+1} + \varepsilon_{z,t} \end{aligned} \tag{1.2}$$

$$\tilde{R}_t = f\left(\tilde{R}_{t-1}, \tilde{\pi}_t, E_t[\tilde{\pi}_{t+1}], \tilde{x}_t, E_t[\tilde{x}_{t+1}]\right) + \varepsilon_{R,t} \tag{1.3}$$

Output, inflation, and the Federal Funds Rate ($\tilde{x}_t$, $\tilde{\pi}_t$ and $\tilde{R}_t$) are expressed as percentage deviations from their steady state values and z_t represents potential output. Eq. (1.3) is the monetary authority's policy rule. This paper considers two different policy rules:

$$\tilde{R}_t = \rho_r \tilde{R}_{t-1} + (1-\rho_r)(\psi_1 \tilde{\pi}_t + \psi_2[\tilde{x}_t - z_t]) + \varepsilon_{R,t} \tag{1.4}$$

$$\tilde{R}_t = \rho_r \tilde{R}_{t-1} + (1-\rho_r)(\psi_1 E_1[\tilde{\pi}_{t+1}] + \psi_2[\tilde{x}_t - z_t]) + \varepsilon_{R,t}. \tag{1.5}$$

Policy rule (1.4) targets current inflation and the current output gap. Policy rule (1.5) targets expected inflation and the current output gap.

This model yields an indeterminate solution if the responsiveness to inflation, ψ_1, is less than one.[2] This condition is valid for both policy rules. See Bullard and Mitra (2002), Woodford (2003), Lubik and Schorfheide (2004), or Evans and McGough (2005), for details regarding indeterminacy. Under indeterminacy, agents' self-fulfilling beliefs influence the model and destabilize the economy. Clarida et al. (2000) estimate Eq. (1.5) to test for indeterminacy. They divide their sample into two periods corresponding to before and after Paul Volker became the Chairman of the Federal Reserve in 1979. Using revised estimates of GDP and deriving expected inflation from

* Tel.: +1 859 456 2577; fax: +1 859 323 1920.
E-mail address: pshea2@email.uky.edu.

[1] For a detailed description and derivation of this model, see King (2000), or Woodford (2003).

[2] This condition is valid for both policy rules. See Bullard and Mitra (2002), Woodford (2003), Lubik and Schorfheide (2004), or Evans and McGough (2005), for details regarding indeterminacy.

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data on the ex-ante real interest rate, they find indeterminacy in the earlier period and determinacy in the latter.

Two approaches have refined the results of Clarida et al. (hereafter "CGG"). The condition that indeterminacy occurs if $\psi_1 < 1$ results from the NK model and is not necessarily valid for different models. Lubik and Schorfheide (2004) therefore estimate the entire model instead of just the policy rule, treating expected inflation as an endogenous rational expectation instead of an exogenous variable. Using a Bayesian approach, they confirm the general result of CGG: indeterminacy prior to Volker's tenure and determinacy afterwards.[3]

CGG, and Lubik and Schorfheide (hereafter "LS") both use ex-post data in their estimations. Orphanides (2001) demonstrates that inflation and output data are often revised after their initial reporting. Using data from internal Fed documents, he also demonstrates that policymakers dramatically overestimated potential output for most of the 1970's and part of the 1980's. The Fed therefore persistently overestimated the magnitude of the output gap. Orphanides (2004) collects real-time data on the Fed's expectations of inflation and re-estimates the model of CGG. He finds determinacy both before and after 1979. Like CGG, however, he only estimates the policy rule and treats expected inflation as an exogenous variable.

This paper follows LS by estimating the entire NK model, but integrates the most striking feature of Orphanides' (2004) data; the Fed's persistent mismeasurement of the output gap. LS use a prior distribution where κ from Eq. (1.2) has a high mean. Section 2 demonstrates that a prior distribution with a lower mean results in indeterminacy during both periods. Section 3 modifies the NK model to allow for mismeasurement of the output gap and tests for indeterminacy using the LS technique. With mismeasurement of the output gap, I find indeterminacy in both periods. Section 4 concludes.

2. The LS results and κ

Macroeconomists disagree on the value of κ. Table 1 summarizes some different values of κ from the literature. It also reports α, the percentage of firms that do not change their price each period in a Calvo-pricing model to generate that value of κ.[4]

To test the robustness of the LS results, I re-estimate the model for two alternate prior distributions of κ.[5] The *low-κ* prior is a gamma distribution with a mean of 0.045 and a standard deviation of 0.01.The *uniform-κ* is a uniform distribution between zero and positive infinity.[6] Given the small sample and large numbers of parameters, it is not surprising that the posterior distribution of κ depends on its prior. Table 2 reports the results for the pre-Volker period for policy rules targeting both current inflation (CIT) and expected inflation (EIT).

Table 2 demonstrates that the LS results in the pre-Volker period are robust to the choice of priors. The results for the latter period, however, do depend on the prior, as reported in Table 3.

The *low-κ* prior suggests indeterminacy and the *uniform-κ* prior places significant likelihood on indeterminacy for both policy rules.

3. Mismeasurement of the output gap

Orphanides (2004) and Orphanides et al. (2000) demonstrate that inflation's measurement error is small compared to that of the output gap. I therefore assume agents accurately observe prices. I also assume that agents accurately observe data at level of the individual household or firm. Firms therefore observe their own output and potential output, as well as that of their competitors. Agents are unable, however, to correctly aggregate data on the output gap. Because utility and profit maximization depend on data for individual agents and not the aggregate output gap, Eqs. (1.1) and (1.2) are unchanged from Section 1. Agents observe an estimate of the output gap, $rtgap_t$:[7]

$$rtgap_t - (\tilde{x}_t - z_t) = \rho_f[rtgap_{t-1} - (\tilde{x}_{t-1} - z_{t-1})] + \varepsilon_{f,t}.$$

The policy rule targeting current inflation becomes:

$$\tilde{R}_t = \rho_r \tilde{R}_{t-1} + (1-\rho_r)(\psi_1 \tilde{\pi}_t + \psi_2 rtgap_t) + \varepsilon_{R,t}$$

Because the output gap appears in Eq. (1.5), expected inflation (with mismeasurement), $E_t^f[\pi_{t+1}]$, differs from its fully rational expectation, $E_t[\pi_{t+1}]$:

$$E_t^f[\tilde{\pi}_{t+1}] = \tilde{\pi}_t/\beta - \kappa rtgap_t/\beta \tag{3.1}$$

$$E_t^f[\tilde{\pi}_{t+1}] = \tilde{\pi}_t/\beta - \kappa(\tilde{x}_t - z_t)_t/\beta \tag{3.2}$$

Using Eqs. (3.1) and (3.2), I write expected inflation as a function of its fully rational expectation:

$$E_t^f[\tilde{\pi}_{t+1}] = E_t[\tilde{\pi}_{t+1}] + \kappa/\beta(\tilde{x}_t - z_t - rtgap_t) \tag{3.3}$$

The policy rule targeting expected inflation then becomes:[8]

$$\begin{aligned}\tilde{R}_t = \rho_r \tilde{R}_{t-1} + (1-\rho)[\psi_1 E_t[\tilde{\pi}_{t+1}] + \psi_1\kappa(\tilde{x}_t - z_t)/\beta \\ +(\psi_2 - \psi_1\kappa/\beta) rtgap_t] + \varepsilon_{R,t}\end{aligned} \tag{3.4}$$

[3] The authors estimate both policy rules but only report their results for the rule that targets current inflation. Both rules yield similar results. The approach of Lubik and Schorfheide (2004) has additional advantages if one is interested in more than just testing for indeterminacy. Indeterminacy has two effects on a system. First, it implies an additional degree of serial correlation. Second, it allows sunspots to affect real variables. The authors' approach allows them to quantify these two effects.

[4] See Woodford (2003) for details on the relationship between the Calvo pricing and k. The value of k depends on several parameters besides $\propto$. Table 1 uses the Woodford (2003) calibration of these other parameters to derive the accompanying values of $\propto$.

[5] LS use a gamma distribution with a mean of 0.5 and a standard deviation of 0.20 as a prior for k.

[6] For both alternate priors, the distributions on all parameters besides k follow LS.

[7] Because households and firms do not need to forecast future interest rates, it does not matter whether they are accurately able to observe the aggregate output gap.

[8] As with the original model, indeterminacy arises if $\psi_1 < 1$.