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Power law and evolutionary trends in stock markets

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Abstract

We model the distribution of daily stock trading volume using the power law and document a new phenomenon. The power law exponent systematically increases with time suggesting that trading is becoming increasingly concentrated in a subset of stocks.

Keywords: Zipf's Law; Hirschman–Herfindahl Index; Stock trading volume

JEL classification: G10; H39; D39

1. Introduction

Trading in stocks has grown dramatically over the last five decades but it is an open question as to whether this growth has been uniform across all stocks or is disproportionately concentrated in a subset of stocks. To measure the unevenness of trading across stocks, we model the distribution of daily trading volume across all stocks in the U.S. market and in each of the three U.S. exchanges as a power law function. We discover a new phenomenon in which the power law exponent systematically increases with time and we find exponents evolve for the market and for each of the exchanges over the years 1962 to 2005. This non-static, evolutionary trend is striking because physical and social science studies that employ the power law consistently report that exponents are stable over time. Prior studies that employ the power law to model financial variables such as firm size (Axtell, 2001), stock returns (Gabaix et al.,

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2003) and net income (Okuyama et al., 1999) report that the exponents are similar across different types of markets and tend to be stable over time. Changes in the power law exponent over time allow us to select from competing hypotheses to describe the evolution of concentration in trading across stocks and we conclude that daily trading is becoming more concentrated in a subset of stocks.

Even casual observation of the financial press shows an explosive growth in stock trading volume in recent decades. New savings opportunities such as Individual Retirement Accounts and 401(K) programs and the explosive growth of financial information have drawn new investors to the market. Stock exchanges, in turn, have responded by expanding the stock offerings available to investors. The effects of these changes on the market may be examined through the lens of Zipf's Principle of Least Effort. This principle holds that there are two opposing forces at play, those of unification and diversification. The influence of these opposing forces suggests three alternative hypotheses. The first hypothesis, consistent with the force of diversification, is that an increase in the number of investors and investment opportunities would increase competitiveness in the stock market with investors spreading their money over a larger set of competitive choices. This implies that investors devote attention and analysis to all stocks and trading is uniformly distributed over all stocks. An alternative hypothesis, consistent with the force of unification, is that, investors, overwhelmed by the plethora of stock investments offered, limit their analyses and choices to a small subset of highly regarded stocks, and in the limiting case the daily choice reduces to a single stock, consistent with herding behaviour. A third hypothesis is that increases in trading volume and investment opportunities are unrelated to market competitiveness leading to purely random fluctuations in market concentration. In this case, forces of unification and diversification offset, leading to a static level of trading concentration. To determine if and how the stock market is evolving, we examine if there are changes over time in a measure of market concentration and discern between these three alternatives.

The power law is widely recognized as a universal law in describing many physical phenomena. Zipf (1949) extended the use of the power law beyond the physical sciences and showed that the law was applicable to the social sciences. A generalization of Zipf's finding, applied to any item that can be ranked by size, can be stated as:

$$(\text{Size})_i \times (\text{Rank}_i)^q = \text{constant} \tag{1}$$

The exponent 'q' is specific to the item examined and is known as Zipf's parameter or the power law exponent. Zipf's Law is a special case of the power law in which the exponent 'q' equals 1. Power law functions and, in some cases, Zipf's Law, have been found to accurately model a remarkably wide range of physical science, social science, and economic phenomena. For example, Gopikrishnan et al. (2000) study the trading of individual stocks over a 2-year (1994–95) period and show that the distribution of trading volume for a stock obeys a power law function with an exponent of approximately 1.5 and that this value seems to hold for each of the 1000 largest stocks. Naldi (2003) shows that the power law exponent can also be used as a general measure of market concentration, and that the larger the exponent the greater the level of concentration.

2. Analysis

In this paper, we model the distribution of the trading volumes for each day across all stocks in the US stock market as a power law function and estimate the value of the daily power law exponent both at the