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The term structure of interest rates under regime shifts and jumps

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Abstract

This paper develops a tractable dynamic term structure models under jump-diffusion and regime shifts with time varying transition probabilities. The model allows for regime-dependent jumps while both jump risk and regime-switching risk are priced. Closed form solution for the term structure is obtained for an affine-type model under log-linear approximations.

Keywords: Term structure; Regime switching; Jump diffusion; Marked point process

JEL classification: G12; E43

1. Introduction

The Fed conducts monetary policy by targeting the short-term interest rate. Many dynamic models of the term structure of interest rates have included a Poisson jump component to reflect the impact of the policy actions.[2] A notable feature of the monetary policy behavior is that those discrete changes in the interest rate target *of the same direction* are very persistent. For example, the Fed decreased the interest rate target 12 times consecutively between January 2001 and November 2002, and since June 2003 there

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[1] Tel.: +1 816 235 5850.
[2] Some recent studies include Ahn and Thompson (1988), Das (2002), Piazzesi (2005) among others.

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have been 11 interest rate hikes by the Fed without a single decrease. Presumably such shifts in the overall monetary policy stance (from accommodative to tightening or vice versa) have more important effects on interest rates than a single interest rate change does.

In this paper we propose a regime-dependent jump-diffusion model of the term structure of interest rates to capture the effects of not only discrete jumps in the interest rate target, but also shifts in the policy regime. The paper is based on a strand of recent studies on term structure models with regime shifts, including Landen (2000), Bansal and Zhou (2002), Dai et al. (2003) and Wu and Zeng (2005) among others. The main contribution of the present paper is that it proposes a simple framework to model both discrete jumps and regime shifts in interest rates.

2. The model

2.1. State variables

We assume that the first L state variables, X_t, are described by the following equation

$$\mathrm{d}X_t = \Theta(X_{t-}, S_{t-})\mathrm{d}t + \Sigma(X_{t-}, S_{t-})\mathrm{d}W_t + J(X_{t-}, S_{t-})\mathrm{d}N_t \tag{1}$$

where S_t is another state variable following a K-regime continuous-time Markov chain to be specified below, X_t and $\Theta(X_{t-},S_{t-})$ are both $L \times 1$ vectors; $\Sigma(X_{t-},S_{t-})$ is a $L \times L$ matrix, W_t is a $L \times 1$ vector of independent standard Brownian motions; N_t is a $L \times 1$ vector of independent Poisson processes with $L \times 1$ time-varying and regime-dependent intensity $\delta_J(X_{t-},S_{t-})$; $J(X_{t-},S_{t-})$ is a $L \times L$ matrix of regime-dependent random jump size with a conditional density $g(J|X_{t-},S_{t-})$. Given $\{X_{t-},S_{t-}\}$, we assume that $J(X_{t-}, S_{t-})$ is serially independent and is also independent of W_t and N_t.

To get a convenient representation of S_t, we define the mark space E as

$$E = \{(i,j) : i \in \{1,\ldots,K\}, j \in \{1,2,\ldots,K\}, i \neq j\}$$

with σ-algebra $\mathcal{E} = 2^E$. Let $z=(i,j)$ be a generic point in E and A a subset of E. A marked point process, $\mu(t, A)$, counts the cumulative number of regime shifts that belong to A during $(0,t]$. $\mu(t,\cdot)$ can be uniquely characterized by its stochastic intensity kernel,[3] which is assumed to be

$$\gamma_\mu(\mathrm{d}t, \mathrm{d}z) = h(z, X_{t-})\mathbf{I}\{S_{t-} = i\}\epsilon_z(\mathrm{d}z)\mathrm{d}t, \tag{2}$$

where $h(z, X_{t-})$ is the regime-shift (from regime i to j) intensity at $z=(i, j)$, $\mathbf{I}\{S_{t-}=i\}$ is an indicator function, and $\epsilon_z(A)$ is the Dirac measure (on a subset A of E) at point z (defined by $\epsilon_z(A)=1$ if $z \in A$ and 0, otherwise). Heuristically, for $z=(i, j)$, $\gamma_\mu(\mathrm{d}t, \mathrm{d}z)$ is the conditional probability of a shift from regime i to regime j during $[t,t+\mathrm{d}t)$ given X_{t-} and $S_{t-}=i$. The compensator of $\mu(t, A)$ is then given by

$$\gamma_\mu(t, A) = \int_0^t \int_A h(z, X_{\tau-})\mathbf{I}\{S_{\tau-} = i\}\epsilon_z(\mathrm{d}z)\mathrm{d}\tau. \tag{3}$$

This simply implies that $\mu(t, A) - \gamma_\mu(t, A)$ is a martingale.

[3] See Last and Brandt (1995) for detailed discussion of marked point process, stochastic intensity kernel and related results.