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Additional properties of a linear pen's parade for individual data using the stochastic approach to the Gini index

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Abstract

The stochastic approach to the Gini index is used to derive new properties of a linear Pen's parade for individual data. With small sample adjustments, the reference value of the Gini index is independent of the number of income-receiving units.

Keywords: Income inequality; Stochastic approach; Gini index; Pen's parade

JEL classification: C81; D31; D63

1. Introduction

Milanovic (1997) derived a simple formula that expresses the Gini index for individual/ungrouped data as a product of a constant ($1/\sqrt{3}$), the coefficient of variation of income, and the correlation coefficient between income and its rank. Using this formula, Milanovic shows that if Pen's (1971, 1973) parade is linear (i.e., income increases by a constant amount as its rank increases by 1 unit), the Gini index will be approximately equal to 1/3, provided that the number of income-receiving units (individuals, households) is sufficiently large. In contrast, a convex Pen's parade, which is formed from a linear parade by transferring income from poorer to richer income-receiving units, yields a Gini index which is greater than 1/3. For a concave Pen's parade, which is formed from a linear parade by transferring income from richer

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to poorer income-receiving units, the Gini index is less than 1/3. In deriving the relevant results, Milanovic assumed that the intercept of the linear regression of income on its rank is close to zero but no restrictions were imposed on the slope.

In this paper, the stochastic approach to the Gini index is used to extend Milanovic's results by deriving additional properties of a linear Pen's parade for individual data without imposing any restrictions on the intercept.[1] It is also well known (e.g., Monti, 1992; Deltas, 2003) that the Gini index is biased downwards. In large samples, which are typically the case in most income distribution studies, this bias is small. However, serious bias problems could arise in small sample situations, such as those typically used when making regional comparisons of inequality or in the measurement of industry concentration. To adjust for this downward bias, the value of the Gini index is multiplied by $n/(n-1)$, where n is the number of income-receiving units, as suggested by Deltas, among others. In this paper, we also show that if small sample adjustments are made, the reference value of the Gini index for a linear Pen's parade will be 1/3 regardless of the number of income-receiving units. However, the Gini index could be less than, equal to, or greater than the reference value depending on whether the intercept is positive, zero, or negative, respectively.

The layout of the rest of the paper is as follows. In Section 2, a brief overview of the stochastic approach to the Gini index for individual data is provided. In Section 3, a generalization of Milanovic's results for individual data is provided. Section 4 concludes.

2. The stochastic approach: individual data

Let y_1, y_2,…y_n be the individual incomes of n income-receiving units, which are arranged such that $y_1 \le y_2 \le \ldots \le y_n$, in which case the rank of the lowest income is 1 and that of the highest income is n. In the case of ties, the incomes are assigned the average of the ranks they would get assuming that they were not tied. Furthermore, let the mean income of the n income-receiving units be denoted by $\bar{y}$.

Using the stochastic approach, Ogwang (2000, 2004) showed that the Gini index could be expressed in two equivalent ways.

First, the Gini index is given by the formula

$$\hat{G}_1 = (2/n)\hat{\theta}-1-(1/n) \tag{1}$$

where $\hat{\theta}$ is the WLS estimator of θ in the model $i=\theta+v_i$ $i=1,2,..,n$ assuming that the errors, v_i, are heteroscedastic of the form $E(v_i^2)=\sigma^2/y_i$.

The second equivalent formula for the Gini index is

$$\hat{G}_2[(n^2-1)/6n]\hat{\beta} \tag{2}$$

where $\hat{\beta}$ is the OLS estimator of β, the modified model $(y_i/\bar{y})=\alpha+\beta_i+\varepsilon_i$, $i=1,2,..,n$ assuming that the errors, ε_i, are homoscedastic i.e., $E(\varepsilon_i^2)=\sigma^2$.

[1] The term "stochastic approach" as used in this paper (and previous papers) involves specifying an underlying regression model for which the error term exhibits a known form of heteroscedasticity, which is related to income. Estimates of the Gini index and its standard error are obtained from the ordinary least squares (OLS) or weighted least squares (WLS) estimates of the parameters of the underlying regression model and those of its standard error, respectively.