Available online at www.sciencedirect.com
ScienceDirect
Economics Letters 95 (2007) 234–240
economics letters
www.elsevier.com/locate/econbase

When different market concentration indices agree

David A. Hennessy [a,*], Harvey Lapan [b]

[a] Department of Economics and CARD, 578C Heady Hall, Iowa State University, Ames, IA 50011-1070, United States
[b] Department of Economics, Iowa State University, United States

Received 21 December 2005; received in revised form 27 September 2006; accepted 11 October 2006
Available online 22 February 2007

Abstract

The Herfindahl–Hirschman, entropy and *N*-firm indices may not agree when comparing market concentrations. For different oligopoly markets, the majorization pre-ordering is shown to be both necessary and sufficient for all to agree.

Keywords: Antitrust; Cost dispersion; Majorization

JEL classification: D4; C6

1. Introduction

The measurement of market concentration is important for several reasons. In many jurisdictions, market concentration indices are used when determining whether a merger should be allowed and whether an existing firm should be broken up. In addition, some companies use market concentration indices when re-organizing production activities. Furthermore, market structure is believed to affect market efficiency in a variety of ways such as altering incentives to innovate (Aghion et al., 2005).

The most widely considered market concentration indices are the Herfindahl–Hirschman index ($H(S)$), the entropy index ($E(S)$), and the k-firm concentration ratio $R(k;S)$, where S is a vector of market shares and we will shortly define each index. If considering N-firm industries, we have just enumerated $N+1$ meaningful concentration indices. Given a vector of market shares, it is possible to construct a second

* Corresponding author. Tel.: +1 515 294 6171; fax: +1 515 294 6336.
E-mail address: hennessy@iastate.edu (D.A. Hennessy).

0165-1765/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.econlet.2006.10.011

vector such that N of these indices agree in ranking but the remaining index does not. We ask the question: when market structure changes then what conditions must be imposed such that all of these indices agree about the consequences of the change for the extent of market concentration? We do so when working directly with share vectors for any pair of markets. We also do so when working indirectly with normalized marketing margin vectors for a pair of Cournot oligopoly markets possessed of constant unit costs.

2. The set of indices

We adopt the following two conventions. Parentheses () are used in subscripts to identify the lower order statistics for a vector, i.e., for $X=(x_1,x_2,\ldots, x_N)$ then $x_{(1)}\le x_{(2)}\le\ldots\le x_{(N)}$. Similarly, square brackets [] are used in subscripts to identify the lower order statistics, i.e., $x_{[N]}\le x_{[N-1]}\le\ldots\le x_{[1]}$. Of course, $x_{(n)}\equiv x_{[N-n+1]}$.[1] For market share vector $S=(s_1,s_2,\ldots, s_N)$ in a N-firm industry, write

$$H(S)=\sum_{n=1}^{N} s_{[n]}^2;\quad E(S)=-\sum_{n=1}^{N} s_{[n]}\mathrm{Ln}[s_{[n]}];\quad R(k;S)=\sum_{n=1}^{k} s_{[n]}. \tag{1}$$

All of these formulae are symmetric in that one could interchange, say, s_i with s_j in the formula without changing the formula value. Function $H(\cdot)$ is convex, while $-E(\cdot)$ is convex. The concentration ratios are not of uniform curvature. Symmetry is an appealing property because it ensures that the index treats the firms identically. Convexity is appealing because it requires that the marginal contribution to the index increases with firm market share, suggesting that a larger firm is of particular concern when market concentration is an issue.

Pairs of vectors, $S'=(s'_{[1]},s'_{[2]},\ldots, s'_{[N]})$ and $S''=(s''_{[1]},s''_{[2]},\ldots, s''_{[N]})$, can be readily constructed to show that all but one of the index set $\{H(S'),-E(S'),R(1;S'),\ldots, R(N-1;S')\}$ is larger than its counterpart in $\{H(S''),-E(S''),R(1;S''),\ldots,R(N-1;S'')\}$. One question we ask is whether a set of conditions for comparing a pair of market share vectors exists such that all of these indices agree about which represents the more concentrated market structure? Another is whether, given Cournot market structure, a set of conditions on cost primitives exists such that the indices concur.

3. Model

We compare behavior across markets A and B. With unit costs across N_i active firms as c_n^i, $n\in\{1,2,\ldots, N_i\}=\Omega_{N_i}, i\in\{A,B\}$, firm outputs as q_n^i, market outputs as $Q^i=\sum_{n\in\Omega_N}q_n^i$, and inverse demand functions as $P^i(Q^i)$, the standard Cournot oligopoly model asserts that firm output choices satisfy

$$P^i(Q^i)+q_n^iP_Q^i(Q^i)-c_n^i=0; \tag{2}$$

where $P_Q^i(Q^i)<0$ is the first derivative.[2] We make the standard assumptions about demand, equilibrium existence and equilibrium uniqueness, and hold that all choices are interior. Optimum choices are characterized as $q_n^{*,i}$.

[1] But the two ways of presenting the same order statistic will prove to be convenient because the order of statistics for a unit cost vector will be the reverse of those for production shares in Cournot oligopoly.

[2] The analysis to follow can be extended to the context of quadratic costs of form $c_n^iq_n^i+0.5\gamma(q_n^i)^2, \gamma>P_Q^i(Q^i)$, where the constraining feature is that γ be firm-invariant.