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Discontinuous extraction of a nonrenewable resource☆

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Abstract

This paper examines the sequence of optimal extraction of nonrenewable resources in the presence of multiple demands. We provide conditions under which extraction of a nonrenewable resource may be discontinuous over the course of its depletion.

Keywords: Backstop technology; Dynamic optimization; Energy resources; Herfindahl principle; Multiple demands

JEL classification: Q3; Q4

1. Introduction

A fundamental result in resource economics, the *Herfindahl rule*, is that when there is a single demand, extraction of identical deposits of a nonrenewable resource should be in the order of their unit costs of extraction (e.g., Herfindahl (1967), Solow and Wan (1976), Lewis (1982)). However, using a model of trash hauling between cities (demands) and landfills with a fixed capacity (resources), Gaudet et al. (2001) prove a "vacillation" result: in the presence of setup costs a city may temporarily abandon a low marginal cost site, move to a higher cost site and then return to the former at a later date. An implication of this result is that a nonrenewable resource may be extracted discontinuously, i.e., over two

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disjointed time periods. In this paper, we show that discontinuous extraction of a nonrenewable resource is still possible, *even without setup costs*. We provide conditions for this discontinuity to occur.

We modify the framework of Chakravorty and Krulce (1994, henceforth CK) who consider two nonrenewable resources, oil (O) and coal (C) and two demands, electricity (E) and transportation (T). We add a third backstop resource (B) with an infinite supply (e.g., solar power).[1] While the assumption of a constant unit extraction cost in CK is retained for each resource (c_i, $i=O, C, B$) we specify conversion costs as both resource- and demand-specific (z_{ij}, $i=O, C, B$; $j=E, T$) so that the net cost of resource i in demand j is $w_{ij}=c_i+z_{ij}$.

The planner maximizes the discounted social surplus W with respect to $q_{ij}(t)$, extraction rates of resource i in demand j:

$$W=\int_0^{\infty}e^{-rt}\left[\sum_j\left(\int_0^{\sum_i q_{ij}}D_j^{-1}(x)dx\right)-\sum_{i,j}\left(c_i+z_{ij}\right)q_{ij}(t)-\sum_i\lambda_i(t)\sum_j q_{ij}(t)\right]dt \tag{1}$$

subject to

$$q_{ij}(t)\geq 0;\,Q_i(t)\geq 0;\quad \dot{Q}_i(t)=-\sum_j q_{ij}(t)$$

where r denotes the discount rate, D_j^{-1} the inverse demand function for j, Q_i (t) the stock of resource i available at time t and $\lambda_i(t)$ the co-state variable for resource i. Define the equilibrium price for demand j as $p_j(t)=D_j^{-1}\left(\sum_i q_{ij}(t)\right)$ and the price of resource i in demand j as $p_{ij}(t)=c_i+z_{ij}+\lambda_i(t)\equiv w_{ij}+\lambda_i(t)$. The necessary and sufficient conditions[2] are

$$p_j(t)\leq p_{ij}(t)\qquad \left(\text{if}\quad <\quad \text{then } q_{ij}(t)=0\right); \tag{2}$$

$$\dot{\lambda}_i(t)=r\lambda_i(t); \tag{3}$$

$$\lim_{t\to\infty}e^{-rt}\lambda_i(t)\geq 0;\ \lim_{t\to\infty}e^{-rt}\lambda_i(t)Q_i(t)=0. \tag{4}$$

Conditions (3) and (4) imply that $\lim_{t\to\infty}Q_i(t)=0$ for nonrenewable resource i, $i=O, C$, and $\lambda_B(0)=\lambda_B(t)=0$ for the backstop resource which is in infinite supply.

2. Optimal extraction sequence

Consider the case in which oil is the cheapest resource for both demands and the backstop is the most expensive. That is,

Assumption.

$$0<w_{Oj}<w_{Cj}<w_{Bj}<\infty,\quad j=E,T. \tag{5}$$

[1] At least three resources are needed for discontinuous extraction with two demands. Amigues et al. (1998) provide a single-demand case wherein the Herfindahl rule is violated but with no discontinuous extraction of a resource.

[2] The proof of sufficiency is essentially the same as in CK, hence suppressed.