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Economics Letters 90 (2006) 200–204

economics letters

www.elsevier.com/locate/econbase

GARCH and irregularly spaced data

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Received 3 March 2003; accepted 13 July 2005
Available online 28 November 2005

Abstract

An exact discretization of continuous time stochastic volatility processes observed at irregularly spaced times is used to give insights on how a coherent GARCH model can be specified for such data. The relation of our approach with those in the existing literature is studied.

Keywords: Volatility; Continuous time model; Exact discretization

JEL classification: C22

Two recent papers, Engle (2000) and Ghysels and Jasiak (1998), proposed different formulations of GARCH models for irregularly spaced data. In this note, we exploit results of Meddahi and Renault (2004) to clarify the advantage of each approach and propose a model that combines these advantages.

In the sequel, we assume that a financial price S_t is observed at irregularly spaced dates $t_0, t_1, \ldots, t_n$, with $0=t_0<t_1<\ldots<t_n$. We denote by x_i, $i=1,\ldots,n$, the ith duration ($x_i \equiv t_i - t_{i-1}$), and by ε_i the continuously compounded return of S_t over the period $(t_{i-1}, t_i]$ ($\varepsilon_i \equiv \log(S_{t_i}) - \log(S_{t_{i-1}})$).

In his simplest volatility model, Engle (2000) assumes that the variable σ_{i-1}^2 defined as

$$\sigma_{i-1}^2 = \frac{h_{i-1}}{x_i}, \text{ where } h_{i-1} = \mathrm{Var}\left[\varepsilon_i | \varepsilon_j, x_j, j \leq i-1; x_i\right] \tag{1}$$

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0165-1765/$ - see front matter
doi:10.1016/j.econlet.2005.07.027

follows a GARCH(1,1)-type equation (Bollerslev, 1986). More precisely, under the assumption

$$E\left[\varepsilon_i|\varepsilon_j,x_j,j\leq i-1;x_i\right]=0, \tag{2}$$

Engle (2000) specifies

$$\sigma^2_{i-1}=\omega+\alpha(\varepsilon_{i-1}/\sqrt{x_{i-1}})^2+\beta\sigma^2_{i-2}. \tag{3}$$

In other words, in order to take into account the unequally spaced feature of the returns, Engle (2000) assumes that the *variance per time-unit*, σ^2_{i-1}, follows a GARCH(1,1) equation.

In contrast, Ghysels and Jasiak (1998) specify a GARCH equation for the *total variance* process $\tilde{h}_{i-1}$ defined by

$$\tilde{h}_{i-1}=\mathrm{Var}\left[\varepsilon_i|\varepsilon_j,x_j,j\leq i-1\right]. \tag{4}$$

However, in order to take into account the unequally spaced feature of the returns, Ghysels and Jasiak (1998) assume a time-varying parameter GARCH equation with

$$\tilde{h}_{i-1}=\omega_{i-1}+\alpha_{i-1}\varepsilon^2_{i-1}+\beta_{i-1}\tilde{h}_{i-2}, \tag{5}$$

where the parameters ω_{i-1}, α_{i-1}, and β_{i-1} are functions of the expected duration ψ_{i-1} defined as $\psi_{i-1}=E[x_i|\varepsilon_j,\ x_j,\ j\leq i-1]$ and a structural unknown parameter. The functional forms adopted by Ghysels and Jasiak (1998) are those derived by Drost and Werker (1996) for a weak GARCH representation (Drost and Nijman, 1993) of a GARCH diffusion model (Nelson, 1990) when observations are equally spaced by a length, say, Δ. For instance, Drost and Werker (1996) show that $\alpha_\Delta+\beta_\Delta=\exp(-\ \kappa\Delta)$ where κ is the mean reverting parameter of the continuous time spot variance process. Therefore Ghysels and Jasiak (1998) assume

$$\alpha_{i-1}+\beta_{i-1}=\exp(-\kappa\psi_{i-1}).$$

It is clear that there are several differences between Engle (2000) and Ghysels and Jasiak (1998) approaches. The first one is in the conditioning information: Engle (2000) considered the variance of the return ε_i given the information $\mathcal{F}^d_{i-1}=\sigma\left(\varepsilon_j,x_j,j\leq i-1;x_i\right)$ while Ghysels and Jasiak (1998) considered the information $\mathcal{G}^d_{i-1}=\sigma\left(\varepsilon_j,x_j,j\leq i-1\right)$. Clearly, under the assumption (2), one has

$$\tilde{h}_{i-1}=E\left[h_{i-1}|\mathcal{G}^d_{i-1}\right]. \tag{6}$$

The second difference is in the GARCH formulation: Eq. (3), for the variance per unit of time, implies a time-varying parameter GARCH equation for the total variance process h_{i-1}:

$$h_{i-1}=\omega x_i+\alpha\frac{x_i}{x_{i-1}}\varepsilon^2_{i-1}+\beta\frac{x_i}{x_{i-1}}h_{i-2}. \tag{7}$$

Therefore, by using (6) and the definition of ψ_{i-1}, one gets

$$\tilde{h}_{i-1}=\omega\psi_{i-1}+\alpha\frac{\psi_{i-1}}{x_{i-1}}\varepsilon^2_{i-1}+\beta\frac{\psi_{i-1}}{x_{i-1}}\tilde{h}_{i-2}+\beta\frac{\psi_{i-1}}{x_{i-1}}\left(h_{i-2}-\tilde{h}_{i-2}\right), \tag{8}$$

which differs from (5) because it is not a GARCH equation (due to the presence of the last term) and the time-varying coefficients involve not only the expected value of x_i, ψ_{i-1}, but also the duration x_{i-1}.