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The method of endogenous gridpoints for solving dynamic stochastic optimization problems

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Abstract

This paper introduces a solution method for numerical dynamic stochastic optimization problems that avoids rootfinding operations. The idea is applicable to many microeconomic and macroeconomic problems, including life cycle, buffer-stock, and stochastic growth problems. Software is provided.

Keywords: Dynamic optimization; Precautionary saving; Stochastic growth model; Endogenous gridpoints; Liquidity constraints

JEL classification: C6; D9; E2

1. The Problem

Consider a consumer whose goal is to maximize discounted utility from consumption

$$\max \sum_{s=t}^{T} \beta^{s-t} u(C_s) \tag{1}$$

for a CRRA utility function $u(C)=C^{1-\rho}/(1-\rho)$.[1]

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URL: http://econ.jhu.edu/people/ccarroll. Software archive at http://econ.jhu.edu/people/ccarroll/EndogenousArchive.zip.

[1] Putting leisure in the utility function is straightforward but would distract from the paper's point.

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The consumer's problem will be specialized below to two cases: a standard micro-economic problem with uninsurable idiosyncratic shocks to labor income, and a standard representative agent problem with shocks to aggregate productivity (the 'micro' and the 'macro' models).[2]

The consumer's initial condition is defined by two state variables: M_t is 'market resources' (macro interpretation: capital plus current output) or 'cash-on-hand' (micro interpretation: net worth plus current income), while P_t is permanent labor productivity in both interpretations.

The transition process for M_t is broken up, for convenience of analysis, into three steps. Assets at the end of the period are market resources minus consumption, equal to

$$A_t = M_t - C_t, \tag{2}$$

and capital at the beginning of the next period is what remains after a depreciation factor $\daleth$ is applied,

$$K_{t+1} = A_t \daleth, \tag{3}$$

where $\daleth = (1-\delta)$in the usual macro notation and $\daleth = 1$ in the micro interpretation.

The final step can be thought of as the transition from the beginning of period $t+1$, when capital K_{t+1} but has not yet been used to produce output, and the middle of that period, when output has been produced and incorporated into resources:

$$M_{t+1} = \overbrace{e_{t+1}\Theta_{t+1}P_{t+1}}^{\equiv L_{t+1}} \mathcal{W}_{t+1} + K_{t+1}\mathcal{R}_{t+1} \tag{4}$$

where W_{t+1} is the wage rate; Θ_{t+1} is an iid transitory shock (e.g., unemployment) normalized to satisfy $E_t[\Theta_{t+n}] = 1 \forall n > 0$ (usually $\Theta_t = 1 \forall t$ in the macro interpretation); and e_t indicates labor effort (or labor supply), which for purposes of this paper is fixed at $e_t = 1$, but in general could be allowed to vary. The disarticulation of the flow of income into labor and capital components is useful in thinking separately about the effects of productivity growth (captured by ΘP) and capital accumulation (K).

Permanent labor productivity (in either interpretation) evolves according to

$$P_{t+1} = G_{t+1}P_t\Psi_{t+1} \tag{5}$$

for a permanent shock that satisfies $E_t[\Psi_{t+n}] = 1 \forall n > 0$ and G_t is exogenous and perfectly predictable (see below for varying interpretations of G).

Defining lower case variables as the upper-case variable scaled by the level of permanent labor productivity, e.g. $a_t = A_t / P_t$,we have

$$a_t = m_t - c_t \tag{6}$$

while with a bit of algebra the state transition becomes

$$m_{t+1} = \underbrace{e_t\Theta_{t+1}}_{\equiv l_{t+1}} \mathcal{W}_{t+1} + \underbrace{(a_t \daleth / G_{t+1}\Psi_{t+1})}_{=k_{t+1}} \mathcal{R}_{t+1}. \tag{7}$$

The interest and wage factors are assumed not to depend on anything other than capital and productive labor input; together with the iid assumption about the structure of the shocks, this implies that the problem has a Bellman equation representation (henceforth boldface indicates functions)

$$\mathbf{V}_t(M_t, P_t) = \max_{C_t} \{u(C_t) + \beta E_t[\mathbf{V}_{t+1}(M_{t+1}, P_{t+1})]\} \tag{8}$$

subject to the transition equations.

[2] Different aspects of the setup of the problem will strike micro and macroeconomists as peculiar; with patience, it should become clear how the problem as specified can be transformed into more familiar forms.