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Nash implementation of the majority rule

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Abstract

Given a society confronting two alternatives, we show that the absolute majority rule is the minimal Nash implementable extension of the relative majority rule.
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1. Introduction

Given a society confronting two alternatives x and y, one possible interpretation of a majoritarian decision is based on the idea of a relative majority so that in deciding on the social ranking of x and y, it is the relative number of x and y supporters which matters. In other words, x is decided to be socially at least as good as y if and only if the number of x supporters is at least as much as the number of y supporters. We know by the characterizations of May (1952), Aşan and Sanver (2002) and Woeginger (2003) that the relative majority rule has various appealing properties. On the other hand, it violates a monotonicity condition which Maskin (1999) showed to be necessary for Nash implementability. Hence the relative majority rule fails to be Nash implementable.

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There is another possible interpretation of a majoritarian decision which is based on the idea of an absolute majority. Here, the social ranking between x and y depends on some absolute number that the supporters of these alternatives must reach. In particular, under the absolute majority rule, x is decided to be socially better than y if and only if the number of x supporters is at least a majority.[1] Austen-Smith and Banks (1999), Yi (2005) and Aşan and Sanver (in press) give characterizations of absolute majoritarianism. In fact, we know from Aşan and Sanver (in press) that the absolute majority rule is Maskin monotonic.

We relate these two interpretations of majoritarianism through the concept of a minimal monotonic extension which is due to Sen (1995) who minimally extends social choice functions to social choice correspondences that are Maskin monotonic. This is a way to measure the extent to which social choice correspondences fail to be Maskin monotonic.[2] We show that the absolute majority rule is the minimal monotonic extension of the relative majority rule.

It is also known that Maskin monotonicity is not sufficient for Nash implementability. So, as pointed by Benoit et al. (2005), minimal monotonic extensions need not be Nash implementable. However, the absolute majority rule satisfies the no veto power condition as well, i.e., an alternative which is considered as a best by everybody but possibly one agent is always an absolute majority outcome. So by the sufficiency theorem of Maskin (1999), it is Nash implementable when there are at least three agents. Hence we are also able to show that the absolute majority rule is the minimal Nash implementable extension of the relative majority rule.

Section 2 gives the preliminaries and Section 3 states the results.

2. Preliminaries

Taking any natural number $n \geq 2$, we consider a society $N=\{1,\ldots,n\}$ confronting a set of alternatives $A=\{a, b\}$. Every $i \in N$ has a complete and transitive preference $R_i \in \{-1, 0, 1\}$ over A.[3] We denote by $R=(R_1,\ldots R_n) \in \{-1, 0, 1\}^n$ an n-tuple of individual preferences, reflecting a *preference profile* of the society. A *social choice rule* (SCR) is a mapping F: $\{-1, 0, 1\}^n \rightarrow \{-1, 0, 1\}$ which assigns a non-empty subset of A to each preference profile.[4]

A SCR F is said to be *Maskin monotonic* if and only if

(i) For any $R, R' \in \{-1, 0, 1\}^n$ such that $R_i \geq 0 \Rightarrow R_i' \geq 0$ for each $i \in N$, we have $F(R) \geq 0 \Rightarrow F(R') \geq 0$ and

(ii) For any $R, R' \in \{-1, 0, 1\}^n$ such that $R_i \leq 0 \Rightarrow R_i' \leq 0$ for each $i \in N$, we have $F(R) \leq 0 \Rightarrow F(R') \leq 0$.

[1] When indifferences are ruled and an odd number of agents is assumed, absolute and relative majoritarianism coincide. For other characterizations of majority rules in various restricted frameworks where the two concepts converge to each other, one can see Maskin (1995), Dasgupta and Maskin (1998), Campbell and Kelly (2000).

[2] While the analysis of Sen (1995) made in an abstract social choice environment with a finite set of alternatives, the concept has been later used in various other environments such as Thomson (1999) who computes the minimal monotonic extensions of certain well-known allocation rules in economic environments; Kara and Sönmez (1996) who use this concept in matching problems; Erdem and Sanver (2005) who compute the minimal monotonic extensions of scoring rules.

[3] We let $R_i=1$ (resp. $R_i=-1$) when agent i prefers a to b (resp. b to a) and $R_i=0$ means that i is indifferent between a and b.

[4] At any $R \in \{-1, 0, 1\}^n$, we interpret F(R) equals $1, -1$ and 0 as the outcome being {a}, {b} and {a, b} respectively. Throughout the paper, we use both interpretations equivalently.

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