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Project selection with sets of mutually exclusive alternatives

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ABSTRACT

We study the problem to maximise the net economic benefit of an investment plan by selecting from a portfolio of candidate projects within a given budget constraint. As is well known, with independent projects the economic efficiency of the entire investment plan is maximised if projects are selected according to their benefit-cost ratio until the budget is exhausted. Often, however, the planning of a project involves a stage where a set of alternative concepts or designs are considered. A best alternative is chosen, and the plan is composed from the pool of all such best alternatives. This procedure violates the assumptions underlying the benefit-cost ratio criterion.

In this paper, we set out the correct criterion to use. A real-life example from Norwegian transport planning is provided to show how the global setting into which the project is going to compete, matters for the selection criterion to be used.

1. Introduction

We study the problem to maximise the net economic benefit of an investment plan by selecting from a portfolio of candidate projects within a given budget constraint. One example would be the national transport plans in countries like Norway and Sweden. Assuming independent projects, i.e. (1) all projects may be selected regardless of which other projects are selected, and (2) their benefits and costs stay the same regardless of which other projects are selected, the economic efficiency of the entire investment plan is maximised if projects are selected according to their benefit-cost ratio until the budget is exhausted. To be exact, this result requires projects to be infinitely divisible, but the divisibility matters only for the last project to be included in the plan, and so is of little consequence if projects are small compared to the budget.

Normally, however, the planning of a project involves a stage where a set of alternative concepts or designs are considered. A best alternative is chosen, and the plan is composed from the pool of all such best alternative solutions. This two-step procedure violates the assumptions underlying the benefit-cost ratio criterion, and in fact, neither the benefit-cost ratio nor the net present value of a project is a valid choice criterion in this case.

In this paper, we set out the correct criterion to use in this case. It is in fact a kind of combination of the net present value criterion and the benefit cost ratio. As we will show, when the budget is comfortably large, it resembles or even becomes identical to the net present value criterion. But the tighter the budget gets, the more will it resemble (or even become equal to) the cost benefit ratio. For instance, in the case where the mutually exclusive alternatives are financing over the budget or by user charges, the acceptable size of the deadweight loss from user charging depends on the how tight the budget is.

It is not the first time this criterion had been proposed. Actually, it was proposed as early as 1955 by Lorie and Savage, and formalised and commented upon by authors such as Weingartner (1963, 1966) and others in the sixties. But it obviously got lost in the subsequent more and more complex development of the capital budgeting literature. We show that the criterion is the solution to a one-period knapsack problem with mutually exclusive project alternatives, and that an approximate solution can be found by a simple iterative procedure, just like Lorie and Savage said.

Until the advent of modern principal-agent theory, it could be argued that strict capital constraints in the private sector simply do not exist, or at least have no good reason to exist, and that the whole capital budgeting literature had lost relevance. Be that as it may, in the public sector strict budget constraints will still be the rule. In fact, budgets are the key instruments to implement democratic decisions and to subject the administration at all levels to democratic control. So for the public sector at least, the Lorie and Savage criterion should still be of interest.

Section 2 prepares for the derivation of the Lorie and Savage criterion in Section 3. This it does by reminding the reader of how the benefit cost criterion is derived: It is the solution to a linear programming problem called the continuous knapsack problem with independent projects. The assumptions underlying this problem are necessary and sufficient conditions for the benefit cost

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criterion to be valid. Changing the assumption of independent projects to projects with mutually exclusive alternatives must produce a different criterion, namely the Lorie and Savage criterion, as shown in Section 3. In Section 4, we illustrate the way this criterion functions in a real life example from Norwegian transport planning. In Section 5, we briefly discuss the situations when the new criterion might be of use and its implication for the possibility of local decisions. Section 6 concludes.

2. The benefit cost ratio

Judging from the HEATCO[1] survey of how cost benefit analysis is practised in 25 European countries (HEATCO, 2005a, 2005b), some confusion still exists about the definition of costs to be used in the benefit cost ratio, about its relationship to the net present value and other commonly used indicators, and about the conditions for its validity as a decision-making tool. Even the HEATCO recommendations themselves (HEATCO, 2006) are plainly wrong when they define costs (to be entered in the denominator of the ratio) as the resource consumption of transport providers and government, and benefits (to be entered in the numerator) as the resource gains of travellers and third parties. This is shown in this section. We also show the necessary and sufficient conditions for the benefit cost ratio to be a valid criterion for project selection.

Nearly all of the countries surveyed in HEATCO report that they combine the benefit cost ratio and the net present value. Many of them provide a clear description of when to use the one or the other, but there seem to be some that use some undefined mix of them. Furthermore, fairly many countries use the internal rate of return to compare projects (a criterion that is not suitable for comparing mutually exclusive options, and that may produce wrong results unless all costs occur before all benefits), or even the payback period (a practise that does not take all relevant costs and benefits into consideration).

Assume that our objective is to maximise the net present value of a plan within a given budget constraint. The candidate projects are assumed to be infinitely divisible and mutually independent. That is, any fraction of the costs of a given project will produce a similar fraction of the benefits, and the costs and benefits of a candidate project are not at all dependent on which of the other projects that are included in the plan. There are no other objectives than maximisation of net present value, and no constraints or conditions other than the given budget constraint. We want to show that the necessary and sufficient condition to achieve our objective under these circumstances is that we select projects in descending order of their benefit cost ratio (with costs defined as net outlays over the relevant public budget) until the budget is exhausted. To keep within the budget, only a fraction of the last selected project can normally be implemented.

2.1. The solution to a linear programming problem

By 'benefits' in this section, we mean the net present value of all monetised impacts of a project, except those impacts that are labelled 'costs'. Let $b=(b_1,\ldots,b_n)$ be the benefits of n candidate projects, some of which are to be chosen to form the plan of a government agency. By the 'cost' of a project we mean the net present value of net payments (expenditure minus revenue) that the agency must incur if these candidate projects are to be included in the plan. Thus a cost to someone else than the government agency is treated as a negative benefit, and the agency's revenue is treated as a negative cost. Let $c=(c_1,\ldots,c_n)$ be the vector of costs of all the candidate projects. We assume there is a constraint a on the net present value of the agency's budgets in the period we consider.

The assumption of such a constraint seems to contradict one of the implicit assumptions of discounting, namely free lending and loaning at the same interest rate. The contradiction is resolved if we assume that the constraint is imposed by a political decision at a higher level of government, as it usually is. Such a decision may make sense even if the margin between the lending and loan rate for the government is very small, because the agency's spending involves not just money, but real resources in short supply.

The n projects are infinitely divisible. That is, if we carry out only a part of a project, as measured by budget outlays, we will always achieve the same part of the project's benefits. This is certainly not always reasonable, but it matters less and less the smaller the projects are as parts of the budget. Let $x=(x_1,\ldots,x_n)$ be the parts of each of the projects that are implemented. Thus $x_j \in[0,1]$ for all x_j. Finally, we assume that all projects are independent of each other, i.e., no element of b and c are functions of x. If this seems to be a problematic assumption in any given case, it can often be solved by forming all possible combinations of the interdependent projects and enter these combinations instead of the interdependent projects themselves. But what we have then are mutually exclusive alternatives, and the rule of Section 3 must be applied.

Projects that do not require any part of the budget can be decided upon separately, and projects that do require a part of the budget but have negative net benefits (benefits minus costs) should always be discarded. Thus we may assume without problems that all elements of b and c are strictly positive and that all elements of b are larger than or equal to their corresponding element of c.

The linear programming problem (LP1) based on these assumptions can now be formulated. Implicitly, it is also assumed that there are no binding restrictions other than the budget on the selection of projects. For example, there is no quantified target for the reduction of climate gas emissions.

$$\text{(LP1)} \quad \max_{\mathbf{x}} \sum_{j=1}^{n} b_j x_j \,\text{s.t.} \sum_{j=1}^{n} c_j x_j \le a \text{ and } x_j \in [0,1] \;\forall j$$

The solution to the problem (LP1) is to arrange the candidate projects after their cost benefit ratio b_j/c_j and select them from the top until the budget is used up. Say that the candidate projects are numbered so that $b_1/c_1 \ge b_2/c_2 \ge \ldots \ge b_n/c_n$. If we select them in the order 1, 2, 3, … and so on, we will ultimately come to a project number r such that the sum of the $r-1$ first costs c is less than the budget a, while the sum of the r first is greater than a. Formally, the solution can be written as Eq. (1).

The formal proof that (1) is indeed the solution requires use of the Simplex method, see any textbook in linear programming. An intuitive argument is this: Assume, contrary to (1), that the solution is to exclude some project with a higher benefit cost ratio b_j/c_j than at least one of the r projects selected by (1). If we take out a small slice of project r and replace it by a similar slice of this excluded project, the objective function must increase. Thus in the optimal solution, all selected projects must have higher benefit cost ratios than any project not selected.

$$x_j = \begin{cases} 1 \text{ for } j = 1, \ldots, r-1 \\ \dfrac{a - \sum_{j=1}^{r-1} c_j}{c_r} \text{ for} j = r \\ 0 \text{ for } j = r+1, \ldots, n \end{cases} \tag{1}$$

[1] HEATCO is the acronym for "Developing Harmonised European Approaches for Transport Costing and project appraisal", a European 6th Framework Programme project.