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Inverse identification of inputs of a random utility based model using optimal control

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ABSTRACT

The random utility based multiregional input–output (RUBMRIO) model is used by several integrated land use and transportation models (ILUTMs) to forecast commodity flows in a region and for spatial allocation of production activities. It makes use of multiregional input–output models which are based on random utility theory. In this work, we use optimal control techniques to find optimal final demand and transportation costs of a RUBMRIO model, that would lead to a desired level of commodity flow and production costs.

At first, the RUBMRIO model is formulated as a discrete time dynamical system. It is shown, using Lyapunov argument, that the fixed point of the dynamical system is input-to-state stable (ISS). Then, a discrete time optimal control problem is formulated with states as commodity flows and production costs, and the final demand and transportation costs as control inputs. An optimization problem is then solved to obtain control inputs that would lead to desired commodity flows and production costs.

Further, the proposed methodology is applied to a numerical example. It is shown that the optimal control based method can achieve user specified commodity flows and production costs up to an acceptable accuracy level.

1. Introduction

Owing to constantly increasing awareness about environmental protection, urban sustainability has become a key policy issue over the years (Nash, 2009). It is well known fact that integrated land use and transportation models play a key role in the context of urban sustainability (Spiekermann & Wegener, 2004; Black, Paez, & Suthanaya, 2002). Integrated land use and transport models (ILUTMs) offer invaluable analysis tools for planners working on transportation and urban projects. Analyzing the behavior of ILUTMs is important for policy makers in order to create sustainable cities for future. Hence, study and analysis of ILUTMs are key towards designing better urban environments (Dutta, Arnaud, Prados, Saujot, & Lefévre, 2012).

A number of ILUTMs, for example MEPLAN and TRANUS, make use of multiregional input–output models combined with random utility theory for allocation of activities and land use (Hunt, 1997; De La Barra, 1989). Since the input–output model varies spatially, between different geographical regions, they are called random utility based input–output (RUBMRIO) models. Typically they combine traditional input–output model with multinomial logit model for calculation of commodity flows and production prices between different regions (Zhao & Kockelman, 2004; Juri & Kockelman, 2004). The RUBMRIO model is usually solved iteratively, by applying a set of algebraic equations (Hunt, 1997). The outputs of the RUBMRIO model are commodity flows and production costs. The inputs are the transportation costs between different regions and the final demand values. Traditional use of RUBMRIO model is to predict the steady state commodity flows and production prices for given values of transportation costs, final demand and other input parameters. Particularly Kockelman et al. have used the model extensively to forecast land use and transportation scenarios in Texas and United States (Du & Kockelman, 2012; Ruiz Juri & Kockelman, 2006). They have also suggested variants of the original RUBMRIO model, in order to accommodate various objectives (Huang & Kockelman, 2008; Zhao & Kockelman, 2004; Juri & Kockelman, 2004). However, a methodology for inverse solution of the RUBMRIO model, where inputs are designed based on desired output values is missing from the literature.

In this work, we have solved an inverse problem to determine the inputs such that the desired output levels are met. The RUBMRIO model is represented as a nonlinear discrete time dynamical system with the states as the outputs of the RUBMRIO model and the controls as the inputs. An optimization problem is then

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solved to find the inputs that minimize the $\mathcal{L}_2$ norm between calculated RUBMRIO outputs and the desired output values.

Discrete time dynamical systems have been used to represent several "real world" systems over the years, ranging from queuing theory (Kleinrock, 1975) to cellular automata (Chopard & Droz, 1998). Several other applications include formal languages, temporal logic, perturbation analysis and computer simulation (Ben-Naoum et al., 1995; Sandefur, 1990). A nonlinear discrete dynamical system is represented as a sequence of states given by $\mathbf{x}(k) \in \Re^n$, where $k = 1, 2, \dots, N$, whose evolution equation is given by,

$$\mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{u}(k)) \tag{1}$$

where $\mathbf{u}(k) \in \Re^m$ is the sequence of inputs to the dynamical system and $f : \Re^n \times \Re^m \mapsto \Re^n$ is a vector of bounded nonlinear functionals. The above system is called *input-to-state stable* (ISS), if the sequence of states $\mathbf{x}(k) < \infty$ given the input sequence $\mathbf{u}(k) < \infty \; \forall \, k = 1, \dots, N$ (Sontag & Wang, 1995; Jiang & Wang, 2001). It ensures that every state trajectory of the discrete system in Eq. (1) will be bounded if the control inputs are bounded. In the current work, we have shown via Lyapunov argument (Robinson, 2013), that the parameters of the RUBMRIO model can be designed such that it is ISS. Further the control inputs are obtained by solving an optimal control problem.

Optimal control deals with the problem of finding a control law for a given dynamical system such that a certain optimality criterion is achieved. An optimal control problem involves solving an optimization problem, which involves optimizing a *cost functional* that is a function of state and control variables, subject to the states and controls satisfying the equations of the dynamical system (Bryson & Ho, 1975). Optimal control has various applications ranging from astrodynamics, where it is used to optimize the entry trajectory of a satellite entering a planet's atmosphere (Singh, Bhattacharya, & Vadali, 2009; Dutta & Bhattacharya, 2010) to quantum mechanics, where it is used to optimize the path of a quantum mechanical system to its final state (Peirce, Dahleh, & Rabitz, 1988). As far as ILUTMs are concerned, researchers have used Bayesian melding, to find optimal parameters in UrbanSim, an ILUTM (Ševčíková, Raftery, & Waddell, 2007). As for models based on RUBMRIO theory, optimization techniques like MLE has been used by researchers to estimate parameters of TRANUS (Dutta, Arnaud, Prados, & Saujot, 2014). However, these work assume that the model involved is static, while estimating parameters. Finding the optimal inputs while taking into account the dynamical nature of RUBMRIO model has largely been ignored.

Supply-driven models, for example the Ghosh model (Dietzenbacher, 1997) exist in literature, which solves for the inputs like demand for a given supply. Hence these models are based on inverse of the *demand-driven* philosophy, on which the RUBMRIO model is based. The inverse identification method using optimal control solves for the demand while satisfying the RUBMRIO equations which are demand-driven. To the author's knowledge such a model is missing from the literature.

This paper has two key contributions. First, it is shown that the RUBMRIO model can be represented as a nonlinear discrete time dynamical system and the fixed point of the resulting dynamical system is ISS. Secondly, an optimal control problem is formulated, to find the inputs of the RUBMRIO model that will optimize a given cost functional. Further, the proposed optimal control methodology is applied to solve for the inputs of a RUBMRIO model that minimizes the $\mathcal{L}_2$ norm of the difference between calculated and desired state values of the outputs.

The rest of the paper is organized as follows: in Section 2 the RUBMRIO model is introduced and further in Section 3, it is shown that the discrete time dynamical system is ISS. In Section 4 the optimal control problem is formulated for various cost functionals. In Section 5 the results of application of the optimal control methodology to test problems is discussed and shown. Finally, in Section 6 conclusions are stated highlighting the future work to be done.

2. Random utility based multiregional input–output model

Input–output theory was originally proposed by Leontief, focusing on a single region's industry interactions with business expenditure (Leontief, 1953). The multiregional input–output (MIO) model extends the original input–output theory to include several regions (Leontief, 1986). The RUBMRIO model has since been developed, combining the MIO model to random utility theory (De La Barra, 1989). As stated earlier, the RUBMRIO model combines traditional input–output model with multinomial logit model for allocation of activities and land use. The final output is obtained by recursively solving a set of nonlinear algebraic equations. An intense mathematical description of the model can be found in Ref. (Zhao & Kockelman, 2004).

Input–output models characterize the interaction between market elements, aggregated into *economic sectors*. The model calculates flow of commodities or services between sectors, giving rise to productions and demand. For a multiregional model, flows can occur between different regions, adding a spatial element. The different regions are called *geographical zones*. If the region of interest is a city, then examples of economic sectors can be *students* or *industry workers* and even *employments*. Examples of geographical zones may be a *neighborhood near an university* or *near a chemical factory*.

The inputs to a multiregional input–output model are seen as the *final demand* for a sector in a given geographical zone and associated *transportation cost* that for a sector to be transported between two zones. From these inputs the intermediate demands are calculated. To meet the intermediate demands, production of services and commodities is required. The *production costs* and the *commodity flow* for a sector in a zone is essentially seen as the output of the model. If the multiregional input–output structure is determined by a multinomial logit model, we have the random utility based multiregional input–output model.

2.1. Economic definition of the variables

Let us consider, $M \in \mathbb{N}$ economic sectors and $J \in \mathbb{N}$ geographical regions. Let x_{ij}^n be the commodity flow of sector n from region i to region j and b_j^n be the price of producing sector n in region j. Also let Y_j^n be the final demand of sector n in region j and d_{ij}^n be the price of transporting one unit of sector n from region i to region j. Hence $n = 1, \dots, M$ and $i, j = 1, \dots, J$. As we will see later, x_{ij}^n and b_j^n are the states and Y_j^n and d_{ij}^n are control inputs of the discrete time dynamical system. developed later in the paper.

The other variables that are germane to the RUBMRIO model are the technical coefficients a_j^{mn} and the multinomial logit dispersion parameter λ^n. a_j^{mn} represents the amount of sector m (in dollars) required to produce an dollar worth of sector n in region j and λ^n is a dispersion parameter for sector n, which is used in the multinomial logit equation of the RUBMRIO model. Typically both $a_j^{mn} > 0, \; \forall \, m, n = 1, \dots, M$ and $j = 1, \dots, J$, and $\lambda^n > 0$, $\forall \, n = 1, \dots, M$. An important point to note is that $\sum_{m=1}^{M} a_j^{mn} < 1$, as the total amount of inputs required to produce a dollar of sector n should be less than a dollar.

2.2. Governing equations

The RUBMRIO model consists of several equations and variables having different economic interpretations. However, for simplicity,