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# Dynamic return-volatility dependence and risk measure of CoVaR in the oil market: A time-varying mixed copula model

Bing-Yue Liu [a,b], Qiang Ji [b,c,*], Ying Fan [d]

[a] Department of Statistics and Finance, University of Science and Technology of China, Hefei 230026, China
[b] Center for Energy and Environmental Policy research, Institutes of Science and Development, Chinese Academy of Sciences, Beijing 100190, China
[c] School of Public Policy and Management, University of Chinese Academy of Sciences, Beijing 100049, China
[d] School of Economics & Management, Beihang University, Beijing 100191, China

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ABSTRACT

This study investigates the risk level in the oil market measured by Value-at-Risk (VaR) and conditional VaR (CoVaR), as well as the dynamic and asymmetric dependence between WTI returns and crude oil volatility index (OVX), by constructing six time-varying mixed copula models. Results show that mixed copula between t copula and the 270-degree rotated Clayton copula is the optimal fitting copula to measure dynamic dependence. The estimated time-varying Kendall coefficients indicate that WTI returns and OVX present negative dependence most of the time. There exists a structural change point of dependence between WTI returns and OVX changes on April 17, 2009, while the dependence characteristics within the subsamples are similar to that in the whole sample, indicating the rationality of our time-varying mixed copula models. Finally, the tests show significant risk spillover from OVX to WTI returns and also asymmetric effects for CoVaRs in response to different upside and downside extreme OVX movements.



## 1. Introduction

The trade-off between risk and return is a fundamental issue in asset pricing theory, which has been given much attention by researchers, market investors and analysts. In general, investors require a larger expected return from a security that is riskier. However, after decades of controversy, no consensus exists among researchers on the relation between risk and return across time. Recently, the volatility implied from options prices is commonly considered as the best volatility forecast and risk measure (Mencia and Sentana, 2013). Thus, interest in the relations between return and implied volatility has increased to provide new evidence to this debate.

During the 2008 global financial crisis, a new implied volatility index (OVX) in the crude oil market was launched by the Chicago Board Options Exchange (CBOE); the index can measure the market's expectation of the 30-day volatility of crude oil prices by applying the well-known CBOE Volatility Index methodology to options on the United States Oil Fund (Liu et al., 2013). This index is the first commodity-based implied volatility index; research on its relations with oil price returns will help investors further improve their trading decisions and will contribute, as well, to the development of energy finance, which is what the current study hopes to achieve.

Previous empirical literature has been focused on the relations between returns and historical volatility in financial markets (León et al., 2007; Wu and Lee, 2015; Chang, 2016), while there is few research in energy market. Recently, return–implied volatility relationship has been received extensive attention in traditional financial markets, in which the stock market implied volatility index (VIX) is widely investigated (Sarwar, 2012; Basher and Sadorsky, 2016). Although much research has been done, no consistent conclusion on positive relations (Fleming et al., 1995; Giot, 2005; Guo and Whitelaw, 2006) or negative relations (Dash and Moran, 2005; Tanha and Dempsey, 2015) has been achieved. Furthermore, the conclusion in the stock market cannot be simply applied to OVX because of the more complicated structure of crude oil options (Ji and Fan, 2016).

Research on OVX in the crude oil market has been a research hotspot in the field of energy finance. Existing research has focused on three aspects. Firstly, the predictive ability of the implied volatility index in the oil market is investigated (Szakmary et al., 2003; Agnolucci, 2009; Lux et al., 2016). Secondly, the spillover effect of the implied volatility

* Corresponding author at: Center for Energy and Environmental Policy Research, Institutes of Science and Development, Chinese Academy of Sciences, China.
*E-mail addresses:* jqwxnjq@casipm.ac.cn, jqwxnjq@163.com (Q. Ji).

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indexes between oil and financial markets is explored (Liu et al., 2013; Maghyereh et al., 2016; Bouri et al., 2017). Thirdly, the interdependence between oil price returns and the implied volatility index is investigated. Only three studies have attempted to answer this problem. Aboura and Chevallier (2013) applied the OLS regression model and found that the inverse leverage effect is the dominant effect that drove the WTI crude oil prices from 2007 to 2011. Agbeyegbe (2015) used OLS regression to find regular feedback and leverage effects between oil price returns and OVX changes, as well as employed the quantile regression copula method to find a negative relationship between contemporaneous OVX and oil returns. Ji and Fan (2016) utilised the time-varying parameter GARCH model and regression analysis to verify the role of OVX as a gauge of investor fear instead of risk preference.

The aforementioned studies mainly focus on a linear relationship, but little information on non-linear dependence and the risk spillovers between OVX and oil price returns is provided. Although the quantile regression copula method was used by Agbeyegbe (2015), only a static relationship was explored. The non-linear time-varying dependence between OVX and oil price return is therefore investigated further from the perspective of risk spillover to extend the current literature. This non-linear dependence can help investors predict well the market risk and improve their portfolio strategy and hedging effectiveness.

The principal contributions of this study are as follows. Firstly, six time-varying mixed copula models (TVMC) are constructed by selecting four copulas. Secondly, the VaR for oil price returns conditional on OVX, as well as the CoVaR for oil price returns conditional on the VaR for OVX, are estimated on the basis of the TVMC models. Then, the dynamic and asymmetric risk spillover effects between oil price returns and OVX are explored. Third, the Kendall coefficients are calculated on the basis of the TVMC model with the use of the Monte Carlo simulation method, which can show the dynamic dependence structure between oil price returns and OVX. Fourth, structural change test of dependence proposed by Dias and Embrechts (2009) is introduced for division of subsamples and to investigate the robustness of our findings. Finally, Brent is selected for comparison of the results with WTI.

The remainder of the paper is organised as follows. Section 2 introduces our methodology of TVMC and the measure of VaR and CoVaR. Section 3 presents the data and empirical results. Finally, Section 4 concludes the paper.

## 2. Methodology

This section mainly models the risk spillover and the dynamic and asymmetric dependence between oil price returns and OVX. The modelling process is divided into three parts. Firstly, the upside and downside VaRs for oil price returns conditional on OVX, as well as the CoVaR for WTI returns conditional on the VaR of OVX, are measured to explore the risk spillover of OVX to WTI returns. Secondly, four copulas that can fit negative dependence are selected to construct six TVMC. Then, the VaR and CoVaR measured above can be estimated. The Monte Carlo simulation method is also used to compute the Kendall coefficient on the basis of the TVMC model, which can measure the dynamic dependence between WTI returns and OVX. Finally, structural change test for dependence is further introduced to detect the jump characteristics of dependence and confirm the robustness of our analysis.

### 2.1. Measurement of VaR and CoVaR

#### 2.1.1. VaR

VaR is a most common measure of market risk used by financial institutions due to its simplicity of calculation. Intuitively, the α%-VaR is defined as the worst loss over a target horizon that will not be exceeded with a given level of confidence $1-\alpha$ (Jorion, 2007). In this section, we measure the upside and downside risks for oil price returns using the VaR for short and long position, respectively.

Let $r_t$ be the log return of the underlying asset at time $t$, and given tail probability $\alpha$, the VaR of a long financial position is related to the lower tail of the return's distribution, i.e. $\Pr(r_t \le -VaR_{\alpha,t}^{long}) = \alpha$, whereas the VaR of a short financial position is related to the upper tail of the return's distribution, i.e. $\Pr(r_t \ge VaR_{\alpha,t}^{short}) = \alpha$. Thus, we define the unconditional market downside risk $VaR_{\alpha,t}^{D} = -VaR_{\alpha,t}^{long}$ and the upside risk $VaR_{\alpha,t}^{U} = VaR_{\alpha,t}^{short}$. If the distribution of crude oil return $r_t$ is continuous, the downside risk $VaR_{\alpha,t}^{D}$ is the $\alpha$th quantile of the return's distribution, whereas the upside risk $VaR_{\alpha,t}^{U}$ is the $(1-\alpha)$th quantile of the return's distribution, i.e.

$$Pr\left(r_t \le VaR_{\alpha,t}^{D}\right) = \alpha, \quad Pr\left(r_t < VaR_{\alpha,t}^{U}\right) = 1-\alpha. \tag{1}$$

Since financial time series may have the characteristics of autocorrelation and conditional heteroscedasticity, we employ ARMA-GARCH-t class model to construct their marginal distribution at time $t$ conditional on observations through time $t-1$, $F_{i,t}(\cdot|\mathcal{F}_{t-1})$, $i=1,2$, as follows.

$$r_{i,t} = \varphi_0 + \sum\nolimits_{j=1}^{2}\varphi_j r_{i,t-j} + \varepsilon_{i,t} + \sum\nolimits_{j=1}^{2}\psi_j \varepsilon_{i,t-j} = \mu_{i,t} + \varepsilon_{i,t}, \tag{2}$$

$$\varepsilon_{i,t} = \sigma_{i,t} z_{i,t}, \quad z_{i,t} \sim i.i.d.t_{\upsilon}, \tag{3}$$

$$\sigma_{i,t}^2 = g\left(\sigma_{i,t-1}, \sigma_{i,t-2}, \cdots, \varepsilon_{i,t-1}, \varepsilon_{i,t-2}, \cdots\right), \tag{4}$$

where $\mu_{i,t} = E(r_{i,t}|\mathcal{F}_{t-1})$, $\sigma_{i,t}^2 = Var(r_{i,t}|\mathcal{F}_{t-1})$.

Through the distribution of standardised residual, we can obtain the conditional distribution of variable $r_{i,t}$, i.e.

$$F_{i,t}(x_i|\mathcal{F}_{t-1};\boldsymbol{\theta_i}) = Pr\left(r_{i,t} \le x_i|\mathcal{F}_{t-1}\right) = Pr\left(\frac{r_{i,t}-\mu_{i,t}}{\sigma_{i,t}} \le \frac{x_i-\mu_{i,t}}{\sigma_{i,t}}|\mathcal{F}_{t-1}\right) = t_{\upsilon}\left(\frac{x_i-\mu_{i,t}}{\sigma_{i,t}}|\mathcal{F}_{t-1}\right). \tag{5}$$

Then, from Eq. (5), we can obtain the oil market downside risk $VaR_{1,t}^{D,\alpha}$ and upside risk $VaR_{1,t}^{U,\alpha}$ as follows.

$$VaR_{1,t}^{D,\alpha} = \mu_{1,t} + \sigma_{1,t}\cdot t_{\upsilon}^{-1}(\alpha), \quad VaR_{1,t}^{U,\alpha} = \mu_{1,t} + \sigma_{1,t}\cdot t_{\upsilon}^{-1}(1-\alpha). \tag{6}$$