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A multifactor stochastic volatility model of commodity prices☆

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ABSTRACT

We propose a novel representation of commodity spot prices in which the cost-of-carry and the spot price volatility are both driven by an arbitrary number of risk factors, nesting many existing specifications. The model exhibits unspanned stochastic volatility, provides simple closed-form expressions of commodity futures, and yields analytic formulas of European options on futures. We estimate the model using oil futures and options data, and find that the pricing of traded contracts is accurate for a wide range of maturities and strike prices. The results suggest that at least three risk factors in the spot price volatility are needed to accurately fit the volatility surface of options on oil futures, highlighting the importance of using general multifactor models in pricing commodity contingent claims.

1. Introduction

Commodity contingent claims play a key role in modern financial markets. Commodity producers and consumers actively use futures and options contracts to hedge their exposures to unpredictable price swings. At the same time, speculative activity in these markets has increased over time, leading to large investment flows from institutional investors and wealthy individuals into commodities, a phenomenon commonly known as financialization (Tang and Xiong, 2012). On the public policy side, there has been increasing pressure to understand whether demand for commodity related contracts affects the behavior of underlying prices (Masters, 2008, 2009). All these factors have created a renewed interest in understanding the dynamics and stochastic behavior of spot prices, and the associated derivative contracts traded in these markets.

In this paper we propose a novel representation of commodity prices that generalizes and nests many models commonly found in the literature, such as Schwartz (1997), Schwartz and Smith (2000), Casassus and Collin-Dufresne (2005) and Cortazar and Naranjo (2006), among many others. In our model, we allow for both the cost-of-carry and the spot price volatility to be driven each by an arbitrary number of risk factors, in a way that is simple and straightforward to implement. Empirically, the model performs well when applied to oil futures and options data, yielding accurate valuations of observed contracts for a wide range of maturities and strike prices. As a consequence, the model is able to explain well-known empirical regularities in option markets such as the dynamics of volatility smiles, as well as the skew in risk-neutral distributions.

Early models of commodity prices such as Schwartz (1997) propose multifactor representations of the convenience yield, but leave the volatility of the spot price constant. While providing a good fit to the observed term-structure of futures prices, these models usually perform poorly when applied to options (Cortazar et al., 2016). As a result, recent studies in the commodities literature have focused in incorporating stochastic volatility into the dynamics of spot prices (see e.g. Chiang et al., 2015; Trolle and Schwartz, 2009b).

Our model generalizes several recent stochastic volatility models such as Chiang et al. (2015) by adding a rich multifactor structure to the spot price variance. Specifically, in our model futures prices are driven by N factors (one factor corresponding to the logarithm of the spot price and the remaining $N-1$ factors modeling its cost-of-carry) while options prices are driven by M additional volatility

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factors. Our specification builds on the general affine diffusion framework of Dai and Singleton (2000), and exhibits unspanned stochastic volatility (USV),[1] providing simple closed-form expressions of commodity futures, and yielding easy-to-compute analytic formulas of European options on futures.

We estimate model parameters using quasi-maximum likelihood and the Extended Kalman Filter (EKF) on a sample of daily WTI oil futures and options from January 2006 until December 2014. Our results reveal that the model achieves an accurate fit of the term-structure of futures prices, the implied volatility surface, and the implied volatility skewness. We test the robustness of the model by comparing in- and out-of-sample calibrations.

Our results suggest that the multifactor structure of the model is crucial in pricing accurately futures and options contracts alike. Empirically, we confirm that only the cost-of-carry factors and the spot price are used to fit futures prices, while the volatility factors only affect the pricing of options contracts, consistent with the USV nature of our model. Finally, our analysis reveals that at least two cost-of-carry and three volatility factors are required to obtain accurate futures and options valuations. Adding a fourth volatility factor improves the pricing of options in periods of market stress.

There is recent literature on stochastic volatility models for commodity prices that we survey in Section 2. Within this literature, the papers closest to ours are Chiang et al. (2015) and Trolle and Schwartz (2009b). We believe that we add to their work. For example, we show in Section 3.3 that the model studied in Chiang et al. (2015) is a restricted version of a specification of ours in which we use three factors to model futures contracts and one factor to explain option prices. Trolle and Schwartz (2009b) propose an USV multifactor model of commodity prices within the Heath et al. (1992) (HJM) framework. We show that we can obtain tractable and general results within the widely used affine-diffusion class of models of the spot price, allowing us to generalize a large body of existing literature by embedding an arbitrary multifactor structure in the stochastic behavior of the variance. Furthermore, we provide simple sufficient conditions that deliver USV in multifactor models of the spot price.

The remainder of the article is organized as follows. Section 2 describes our model in its most general form, studies broad sufficient conditions that deliver USV, and reviews the literature. Section 3 explains the affine diffusion implementation of our model, and derives formulas for pricing commodity contingent claims. Section 4 presents the empirical methodology, while results are reported and discussed in Section 5. Section 6 finally concludes. All proofs and details on the numerical estimation are presented in the Appendix.

2. General USV model formulation

We present our model of commodity prices in its most general form, and identify simple, although broad, sufficient conditions that deliver USV. For commodities, such models yield simple valuation formulas for futures prices, while at the same time allowing for arbitrarily complex dynamics in the volatility, which is relevant when pricing options and other derivatives with convex payoffs. On the other hand, there is a large body of literature that has studied general multifactor models of commodity prices while either leaving the volatility constant, or allowing the variance to be driven by a simple univariate process. We show how to naturally embed these well-known models of commodity prices within the more general USV class.

Throughout this paper we consider a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and a filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ generated by standard $\mathbb{P}$-Wiener processes $(\mathbf{W}_t)_{t\geq 0}$ in $\mathbb{R}^{N+2M}$ and satisfying the usual conditions (see e.g. Protter, 2005). The spot price S is described by the process:

$$\frac{dS_t}{S_t} = (y_t + \pi_t)dt + \sigma_S dB_t + \sqrt{v_t}dZ_t, \tag{1}$$

where $(B_t)_{t\geq 0}$ and $(Z_t)_{t\geq 0}$ are standard $\mathbb{P}$-Wiener processes in $\mathbb{R}$ spanned by $(\mathbf{W}_t)_{t\geq 0}$, y represents the cost-of-carry, π designates the commodity risk-premium, and σ_S denotes the constant component of the variance while v denotes its stochastic component. Under the pricing measure $\mathbb{Q}$, equivalent to the physical measure $\mathbb{P}$, the spot price S is described by the process:

$$\frac{dS_t}{S_t} = y_t dt + \sigma_S dB_t^{\mathbb{Q}} + \sqrt{v_t}dZ_t^{\mathbb{Q}}, \tag{2}$$

where $B^{\mathbb{Q}}$ and $Z^{\mathbb{Q}}$ are standard $\mathbb{F}$-adapted $\mathbb{Q}$-Wiener processes in $\mathbb{R}$.

Eqs. (1) and (2) capture the essence of our modeling approach. In the next section we show how to operationalize the model and write the $\mathbb{F}$-adapted processes $(y_t)_{t\geq 0}$, $(\pi_t)_{t\geq 0}$ and $(v_t)_{t\geq 0}$ as multifactor affine diffusions, but for the moment we leave them unspecified. Nevertheless, it will prove useful in our analysis to put some restrictions on the statistical relation between y, v, $B^{\mathbb{Q}}$ and $Z^{\mathbb{Q}}$ in order to (i) obtain simple futures and option valuation formulas, and (ii) separate the problem of fitting futures and option prices. Since these two objectives are achieved when the model exhibits USV, we introduce the following sufficient (although not necessary) assumption that yields the result.

Assumption 1. The $\mathbb{F}$-adapted processes $\left\{(y_t)_{t\geq 0}, \left(B_t^{\mathbb{Q}}\right)_{t\geq 0}\right\}$ are $\mathbb{Q}$-independent of $\left\{(v_t)_{t\geq 0}, \left(Z_t^{\mathbb{Q}}\right)_{t\geq 0}\right\}$.

Notwithstanding its generality and simplicity, we must note that Assumption 1 is not necessary to obtain USV. In Appendix A we present an example of a model that exhibits USV but in which Assumption 1 is violated since the convenience yield is correlated with the stochastic component of the variance. However, as will be shown later in our empirical analysis, the model written using Assumption 1 is already flexible enough to fit futures and option prices well. Hence, we do not find necessary to complicate the analysis further.

Consider now a futures contract $F_{t,\tau}$ at instant t with delivery at time $T = t + \tau$. It is well known that $F_{t,\tau} = \mathbb{E}_t^{\mathbb{Q}}[S_T]$ (Duffie, 2001; Pozdnyakov and Steele, 2004). A direct application of Itō's Lemma allows us to write:

$$\begin{aligned} S_T =& S_t \exp\left\{\int_t^T \left(y_u - \frac{1}{2}\sigma_S^2\right)du + \int_t^T \sigma_S dB_u^{\mathbb{Q}}\right\} \\ &\times \exp\left\{\int_t^T \left(-\frac{1}{2}v_u\right)du + \int_t^T \sqrt{v_u}dZ_u^{\mathbb{Q}}\right\}, \end{aligned} \tag{3}$$

which implies that

$$F_{t,\tau} = S_t \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{\int_t^T \left(y_u - \frac{1}{2}\sigma_S^2\right)du + \int_t^T \sigma_S dB_u^{\mathbb{Q}}\right\}\right] \tag{4}$$

[1] The phenomenon of USV was initially studied in fixed-income markets (Collin-Dufresne and Goldstein, 2002; Li and Zhao, 2006), where it refers to the fact that bonds alone are unable to hedge interest-rate volatility risk, making interest-rate options non-redundant assets. For commodities, USV implies that futures and other commodity linear contracts are unable to hedge spot price volatility risk, making options on futures non-redundant assets.