Energy Economics 63 (2017) 51–65

Contents lists available at ScienceDirect

Energy Economics

journal homepage: www.elsevier.com/locate/eneco

Electricity price modeling with stochastic time change

Svetlana Borovkova[a,*], Maren Diane Schmeck[b]

[a] Vrije Universiteit Amsterdam, The Netherlands
[b] Bielefeld University, Germany

ARTICLE INFO

Article history:
Received 9 November 2015
Received in revised form 8 November 2016
Accepted 6 January 2017
Available online 18 January 2017

JEL classification:
Q41
C13
C22
C51

Keywords:
Electricity prices
Stochastic time change
Activity rate
Mean reversion
Jump diffusion

ABSTRACT

In this paper, we develop a novel approach to electricity price modeling, based on the powerful technique of stochastic time change. This technique allows us to incorporate the characteristic features of electricity prices (such as seasonal volatility, time varying mean reversion and seasonally occurring price spikes) into the model in an elegant and economically justifiable way. The stochastic time change introduces stochastic as well as deterministic (e.g., seasonal) features in the price process' volatility and in the jump component.

We specify the base process as a mean reverting jump diffusion and the time change as an absolutely continuous stochastic process with seasonal component. The activity rate of the stochastic time change can be related to the factors that influence supply and demand. Here we use the temperature as a proxy for the demand and hence, as the driving factor of the stochastic time change, and show that this choice leads to realistic price paths. We derive properties of the resulting price process and develop the model calibration procedure. We calibrate the model to the historical EEX power prices and apply it to generating realistic price paths by Monte Carlo simulations. We show that the simulated price process matches the distributional characteristics of the observed electricity prices in periods of both high and low demand.

1. Introduction

1.1. Motivation

Electricity prices from liberalized markets exhibit features that are rarely observed in other commodity markets. Besides strong demand-related price seasonalities, electricity prices exhibit large spikes, which arise due to non-storability of electricity, non-elasticity of demand and, in case of renewable electricity generation, of supply. The occurrence of positive price spikes is related to supply interruptions and to the increased demand during periods of abnormally high or low temperatures. Negative spikes — which is a rather recent phenomenon — are related to power generation by e.g., wind farms, which cannot be "switched off". Such price spikes are usually short-lived as prices rapidly return to their "normal" levels once disruptions in supply are resolved. The electricity price volatility is also related to demand, increasing during periods of high demand. This leads to seasonal patterns not just in prices, but also in the volatility. All these features make electricity price modeling a challenging task.

A large variety of electricity price models have been described in the literature. The so-called *reduced-form models* range from jump-diffusions or Levy-driven diffusions (Geman and Roncoroni, 2006, Cartea and Figuera, 2005, Meyer-Brandis and Tankov, 2008, Klüppelberg et al., 2010, Barndorff-Nielsen et al., 2013) to regime switching (Huisman and Mahieu, 2001, Paraschiv et al., 2015) to time series models of the ARMA-GARCH type (Benth et al., 2014). Neural networks, agent-based models, fuzzy systems and other AI-based models also have been extensively applied to electricity prices. An important special class of models — the so-called *structural models* — incorporate external factors such as demand, capacity, load and fuel prices into the electricity price formation process. Weron (2014) provides an excellent recent survey of literature on electricity price modeling and forecasting.

Despite the voluminous literature on modeling electricity prices, there is no clear "winner" model. Such model should be versatile enough to generate important price characteristics (seasonalities, spikes and mean reversion), should incorporate at least some supply- and demand-related information, while it should be tractable enough to be useful in applications such as pricing of derivatives (electricity futures and options). In this paper, we suggest such a tractable continuous time model. This model can incorporate demand and supply proxies as the main driving price factors in a new

* Corresponding author.

http://dx.doi.org/10.1016/j.eneco.2017.01.002
0140-9883/

and ingenious way. Our approach is based on the powerful technique of stochastic time change, previously successfully used in modeling other asset prices. The main goal of this modeling exercise is generating price paths with the correct stylized facts and distributional properties, particularly during different seasons (and hence, periods of different demand). This is useful for scenario simulations or any other applications where Monte Carlo generation of realistic price paths is required.

1.2. Stochastic time change

The concept of time-changing of a continuous time stochastic process is a powerful tool for building models in financial mathematics. It allows the model builder to introduce jumps and stochastic volatility into standard models based on the Brownian motion.

Clark (1973) was the first author who observed that prices in financial markets are more volatile on days when a lot of trading takes place. This observation lead to the idea that calender time might not always be the most appropriate way to measure time: in equal calender time intervals, the activity in the market can be very different. On the other hand, we can define a non-equidistant time grid, such that in each time interval the amount of trading activity is the same (the resulting new time scale is said to evolve in *operational time* or *business time*). Consequently, asset returns in each such interval should have the same distributional properties. This has been empirically confirmed by Ané and Geman (2000), who extended Clark's ideas and recovered normality of asset returns by stochastic time change based on order flow.

To define business time, the most important issue is to identify the triggers that determine the speed of the market. For example, Clark (1973) considers the accumulated traded volume as the relevant factor for cotton markets, while Ané and Geman (2000) conclude that in stock markets the appropriate time scale should be defined using the accumulated number of trades.

Stochastic time change can be defined in two distinctive ways. The first one is by means of the so-called *subordinators*, which are non-decreasing Levy processes (pure jump processes plus a linear deterministic drift). Time-changing a Brownian motion by such a subordinator results into another Levy process and hence, introduces jumps into an otherwise continuous stochastic process.

Another way to define it is by means of an absolutely continuous time change. Such time change is defined as the time integral over the so-called *activity rate*, which can be seen as a proxy for the (trading) activity in the market. On a day with a high trading activity, the activity rate is high as well and so, time evolves quicker.

In electricity markets, the spot prices are driven largely by demand, which in turn depends on the outside air temperature. In the summer months, electricity is used for air conditioning and in the winter for heating, which leads to an increased demand for electricity during periods of sufficiently high or low temperatures. A number of previous electricity price studies explore this relationship. For example, Bessec and Fouquau (2008) investigate the effect of temperature on electricity demand in Europe. As this effect depends on the regional climate as well as on heating and cooling habits in different countries, they differentiate between cold, medium and warm countries. In order to filter out the part of the demand that can be explained by the temperature, they remove the effect of other, non-climatic factors on electricity consumption: demographic and technological trends, monthly seasonality (in particular, the decrease in production during summer). They find that the functional relationship between the electricity demand and temperature has a parabolic form, with the minimum at around 16 °C: the "neutral" temperature with respect to energy demand, where neither heating nor airconditioning is needed in continental Europe.

There is also a well-documented positive relationship between energy price volatility and demand. Kanamura (2009) investigates this relationship for natural gas prices and explicitly models gas price volatility as a function of demand. Yang et al. (2002) study this relationship for crude oil prices in US, and a recent study by Jobling and Jamasb (2015) extend this to worldwide oil markets. Li and Flynn (2004) document the relationship between volatility and demand for various electricity markets.

In this paper, we specify an activity rate for the time change in electricity markets on the basis of a demand proxy, which is in our case the temperature. For that, we will explore the functional relationship found by Bessec and Fouquau (2008). However, this is not the only possible choice, and other choices for stochastic clock's activity are possible, for example, those related to the supply (such as wind speed or sunny hours for markets which are largely driven by the wind or solar energy).

When specifying a time-changed stochastic process, we need to decide on the general, basis features of the price evolution. Ané and Geman (2000) show that asset returns are normally distributed, when these are recorded not in calender time, but in business time. Other applications of time-changed processes in finance also try to recover normality in returns. These studies assume a (geometric) Brownian motion as the main driver of the price process in business time, so a Brownian-based diffusion often functions as the basis model for the stock price. This seems to be an important element of the success of those studies. However, electricity prices exhibit more complex features than stock prices, so a Brownian motion alone — neither in an arithmetic nor geometric form — would be appropriate as the basis model. Mean reversion and jumps should be inherent features of such a model. Therefore we choose a mean reverting jump diffusion as the price process in business time. Note that it is possible to introduce jumps into a continuous process (such as Brownian motion) by a Levy subordinator-based time change (but not by an absolutely continuous time change — see Barndorff-Nielsen and Shiryaev, 2010). However, our main goal is to mimic demand-driven seasonal and stochastic features in the price volatility and the jump components, which is possible to do by specifying the absolutely continuous time change with the activity rate related to the demand proxy. Combining mean reverting jump diffusion (the base process) with a stochastic and seasonal activity rate, our resulting spot price model will keep the properties of mean reversion and jumps, but will have stochastic and seasonal features in the jump intensity, rate of mean reversion and volatility.

In the existing finance literature, most studies that consider time changed models use a (geometric) Brownian motion or a general Lévy process as the base process (see e.g., Carr et al., 2003, Carr and Wu, 2004 or Kallsen and Muhle-Karbey, 2011). One exception is Li and Linetsky (2014), who combine a mean reverting model for commodity prices with a time change and derive futures and option prices in terms of Hermite expansions. They examine whether their spot price model leads to futures and option prices with empirically observed features as the Samuelson Effect for futures and implied volatility smiles for options. Lorig (2011) considers mean-reverting models with stochastic volatility, based on Fouque et al. (2000). However, the spot price model assumed in that paper is not itself mean reverting. As Li and Linetsky (2014), Lorig (2011) performs a time-change and, using spectral theory and singular perturbation techniques, derives an approximation for the price of a European option.

In this paper we apply a stochastic time change technique to electricity price modeling, as well as outline a calibration procedure and the empirical application to the German electricity market. Using a demand-based time change, we are able to have a solid economic interpretation of our model. Using the temperature (which can be accurately forecasted) as the proxy for the demand, the model can be applied to obtain distributional forecasts of the electricity price, by predicting its volatility and the probability of a large price spike, which is valuable especially during a cold season.