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Pricing of forwards and other derivatives in cointegrated commodity markets

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ABSTRACT

We analyze cointegration in commodity markets, and propose a parametric class of pricing measures which preserves cointegration for forward prices with fixed time to maturity. We present explicit expressions for the term structure of volatility and correlation in the context of our spot price models based on continuous-time autoregressive moving average dynamics for the stationary components. The term structures have many interesting shapes, and we provide some empirical evidence from refined oil future prices at NYMEX defending our modeling idea. Motivated from these results, we present a cointegrated forward price dynamics using the Heath–Jarrow–Morton approach. In this setting, the concept of cointegration is extended to what we call *cointegration in the limit*, which is an asymptotic form of the notion. The Margrabe formula for spread option prices is shown to hold, with an explicit plug-in volatility. We present several numerical examples showing that cointegration leads to significantly cheaper spread options compared to the complete market case, where cointegration disappears with respect to the pricing measure.

1. Introduction

In this paper we investigate pricing of forwards and other derivatives in a multiple commodities framework with cointegration. In financial economics, the standard modeling choice for the joint stochastic dynamics is correlated geometric Brownian motion (see e.g. Merton, 1973; Margrabe, 1978; Stulz, 1982 for early contributions). Geometric Brownian motion is a non-stationary process and the spread of two correlated geometric Brownian motions will have infinite variance as time approaches infinity. On the other hand, when commodities are cointegrated there exists a linear combination of (log) prices which becomes stationary. The individual commodities may be non-stationary, but there exists a stationary long term linear relationship between them.

We show how cointegration carries over from the spot market to forward contracts. In particular, based on a parametric class of pricing measures and a cointegrated factor model for the spot price dynamics, we show that forward contracts with fixed time to maturities preserve the cointegration property. Moreover, cointegration directly impacts the correlation term structure in the forward market. Inspired by these results, we suggest a Heath–Jarrow–Morton model for the forward market, where a concept of *cointegration in the limit* appears naturally. We apply our modeling and analysis to pricing of spread options, and discuss the impact of cointegration on these derivatives.

There exist two main modeling approaches for contingent claim valuation in commodity markets; spot price models and forward curve models. In a spot price model the starting point is the specification of the stochastic dynamics of the underlying commodity. The unobservable (net) convenience yield plays the same role as a dividend yield for common stocks, since it benefits the spot commodity holder but not the holder of a derivative asset. After an appropriate change of probability measure, forward, futures and (real) option prices can be computed as conditional expectations of the underlying spot price under the pricing measure. Examples of this approach can be found in Brennan and Schwartz (1985), Gibson and Schwartz (1990) and Schwartz (1997). The main problem with spot price based models is that forward prices are given endogenously from the spot price dynamics. As a result, theoretical forward prices will in general not be consistent with market observed forward prices. As a response to this, a line of research has

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focused on modeling the evolution of the whole forward curve using only a few stochastic factors taking the initial term structure as given. Examples of this research, building on the modeling framework of Heath, Jarrow and Morton (HJM) Heath et al. (1992), are Cortazar and Schwartz (1994), Clewlow and Strickland (1999a), Clewlow and Strickland (1999b) and Miltersen and Schwartz (1998). Empirical investigations in commodity markets have been conducted by, among others, Cortazar and Schwartz (1994), Clewlow and Strickland (2000) and Casassus and Collin-Dufresne (2005).

Recently derivatives pricing in cointegrated commodity markets have produced results that, at first, seem inconsistent. For instance, Duan and Theriault (2007) consider a cointegrated forward curve approach in the market for crude oil and oil products. In their model, cointegration has no effect on cross commodity option valuation. The long term stationary relationship disappears in the transition from the real world probability measure to the pricing measure. On the other hand, Casassus et al. (2013) develop an equilibrium model where spot prices of crude oil and oil products are cointegrated through linkages in the convenience yields. In their framework cointegration is preserved after changing from the real world to the pricing measure. The two approaches give very different valuation results for (long term) spread options. We remark that the bivariate GARCH model of Duan and Theriault (2007) is empirically tested against a univariate GARCH approach in a recent paper by Mahringer and Prokopczuk (2015).

The purpose of this paper is to develop a rigorous and coherent modeling framework for cointegrated commodity markets. We do this both in the spot price framework and in the HJM framework. We present some new insights and reconcile results from previous literature in this area.

Starting with a spot price framework, we propose a generalized two-factor model similar to the short-term/long-term commodity model of Schwartz and Smith (2000). The log commodity spot price dynamics consists of two separate processes; a stationary (short term) factor and a non-stationary (long term) factor. We model the stationary factor as a continuous time autoregressive moving average (CARMA) process, and the non-stationary process as an arithmetic Brownian motion. These stochastic processes are analogous to the short term and the long term factors in the Schwartz–Smith model respectively, however, the Ornstein–Uhlenbeck dynamics for the short term factor in the Schwartz–Smith model is replaced by a general CARMA dynamics. The non-stationary process is common to both commodities, while the stationary CARMA processes are specific to each commodity.[1] For our joint dynamics there exist a stationary, linear relationship between the log prices in which the common non-stationary process cancels out. Our spot price model does not explicitly consider stochastic convenience yields.[2] Nevertheless, as shown by Schwartz and Smith (2000), if the short term dynamics is governed by an Ornstein–Uhlenbeck process (which corresponds to a CARMA(1,0) model in our framework) the model is equivalent to the stochastic convenience yield model developed in Gibson and Schwartz (1990); the state variables in each model can be represented as linear combinations of the state variables in the other.

Schwartz and Smith (2000) argue that their model specification of stochastically evolving short-term deviations and long term equilibrium prices seems more natural and intuitive than the stochastic convenience yield set up. Adding to that, the Schwartz–Smith model approach is better suited for modeling non-storable "commodity" markets, like electricity, weather or freight rates, where the convenience of holding inventory makes little sense. Schwartz and Smith (2000) also note that these factors are more "orthogonal" in their dynamics, which leads to more transparent analytical results. This is especially true when generalizing to multiple cointegrated commodity markets. The long term factor must be common to all commodities, while the short term factor can be idiosyncratic to each commodity. Note also that this model is suitable for other asset classes as well. For instance, Fama and French (1988) proposed a discrete time model for the stock price dynamics similar to the Schwartz–Smith model. This means that our framework can also be used for derivatives pricing in cointegrated stock markets as well.

Cointegration is a real world phenomenon and therefore defined under the market (objective) probability measure. For the purpose of derivative valuation, we change the probability measure to the pricing measure. There are two main approaches. The first approach assumes that the commodity itself is a traded asset similar to common stock in the Black–Scholes model (see for instance Brennan and Schwartz, 1985). Derivatives can be replicated by dynamic trading in the underlying commodity, and there exists a unique pricing measure for the commodity. In the second approach the spot price plays the role of an underlying state variable upon which contingent claims can be written. In this latter approach the pricing measure can only be identified after an additional assumption regarding the market price of risk. The market price of risk is typically assumed to have a functional form that makes the spot price dynamics qualitatively similar under the real world and the pricing measure (see for instance Gibson and Schwartz, 1990; Schwartz, 1997; Schwartz and Smith, 2000; Casassus and Collin-Dufresne, 2005).[3]

This basic market assumption is crucial when it comes to derivatives valuation in cointegrated commodity markets. When assuming perfect tradability in the underlying spot commodity, forward prices can be replicated by a simple buy-hold strategy, and thus, the volatility of forwards is equal to the volatility of spot prices. Both the short term and the long term factor of the spot price has the same effect on all parts of the forward curve, and the mean reverting property of the short term factor is not impacting the forward prices in the sense of an exponential dampening (by the speed of mean reversion) of the volatility term structure resulting from the short term factor. Cointegration disappears under the pricing measure and the joint spot (and forward) dynamics reduces to correlated geometric Brownian motions. On the other hand, using the state variable approach, the non-stationary long term factor and the mean reverting short term factor both exist after adjusting for the price of risk. This way spot commodity prices remain cointegrated also under the pricing measure. This explains the fundamental difference between the model of Duan and Theriault (2007)[4] and Casassus et al. (2013).

In our analysis we proceed with the state variable approach and assume constant market prices of risks.[5] We derive several implications for the forward price dynamics and cross-commodity option pricing when cointegration is preserved under the pricing measure. As is known, for a given forward contract the maturity time is fixed, but time to maturity decreases as we move forward in time. We show that the dynamics of two commodity forward contracts are not cointegrated even though the underlying spot prices are.[6] Rolling a contract forward

[1] Paschke and Prokopczuk (2009) propose a slightly different cointegrated spot price model than the one we present here. In their model commodity prices are driven by an n-dimensional log price dynamics. The non-stationary factor is an arithmetic Brownian motion, while the n-1 stationary components are all correlated Ornstein–Uhlenbeck processes. They estimate the model using the Kalman filter approach on 3 commodities (crude oil, heating oil and gasoline) using a 6 factor model.

[2] Nakajima and Ohashi (2012) also consider cointegration through the convenience yield, but they state their model directly under the pricing measure.

[3] See Secomandi and Seppi (2014) for a nice discussion on risk neutral pricing with different commodity market assumptions. They use the term *dynamically complete market* when the commodity itself is a tradeable asset and the market is perfect in the Black–Scholes sense. They argue that this approach applies to precious metals, like silver and gold, which are investible stores of value over time. They use the term *dynamically incomplete markets* when there is non-traded randomness in the spot price dynamics, and the market price of risk approach applies. This latter approach is relevant for most commodities other than precious metals.

[4] The approach of Duan and Theriault (2007) is in fact an application of the model by Duan and Pliska (2004) developed for the stock market. They set up a model for cointegrated stock prices allowing for GARCH-type stochastic volatility. They found that although cointegration disappeared under the pricing measure, the joint stock price dynamics still had non-trivial GARCH effects.

[5] For previous studies assuming constant market price of risks see e.g. Gibson and Schwartz (1990), Schwartz (1997)) and Schwartz and Smith (2000).

[6] Forward contracts in the long end of the market will move with a fixed distance respective to each other on the log scale. Hence, long term forward contracts in a cointegrated set up will stay close to each other, and in this sense they are asymptotically cointegrated.