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On the use of the moment-matching technique for pricing and hedging multi-asset spread options ☆,☆☆,★

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ABSTRACT

The aim of this paper is to show the benefit of applying a moment matching technique to the short leg component in order to price and hedge multi-asset spread options: in particular, we approximate the real dynamics of the short leg component by taking a log-normal proxy, whose equivalent volatility can be computed by performing a two-moment matching approximation. The pricing of the option is then performed once the equivalent correlation parameter between the long leg underlying and the proxy short leg component has been calculated. The main advantage associated with the moment matching approach proposed in this paper is a reduction of the dimension of the pricing problem: we can, indeed, continue using all the option formulas available in the literature for two-legged spread options, i.e. spread options written on two underlyings. Besides it, the combined use of an option formula for two-legged spread options and the moment matching technique applied to the short leg component provides a good approximation to the Monte Carlo simulation. It is well-known that the Monte Carlo price and Greeks can be considered as the benchmark since no exact formula is available for the pricing and hedging of multi-asset spread options. The accuracy of our approach is even comparable to the one provided by using closed form approximation formulas based on three underlyings, where each underlying entering into the short leg component is treated separately.

1. Introduction

Spread options are widely traded both on organized exchanges and over the counter in equity, fixed income, foreign exchange and commodity markets. They play an increasingly important role in hedging correlation risks among a set of assets of concern. In energy markets, for example, clean spark/dark spread options, and their variants designed for exchanging one or several types of fuel for electricity, are commonly used in hedging both short-term and long-term cross-commodity risks.

Moreover, there is a growing demand for spread options involving three assets in bulk quantity with contract parameters spanning a large range. Such scenarios arise from the application of valuing physical assets as spread options. In the following, we will refer to simple-asset spread options to denote two-legged spread options, i.e. spread options written on two underlyings, whereas we will use the term multi-asset spread options to denote spread options written on more than two (normally three) underlyings.

If we consider the case of simple-asset spread options, a number of research works, such as Ravindran (1993), Kirk (1995), Carmona and Durrleman (2003), Deng et al. (2008) and Bjerksund and Stensland (2011), have provided very accurate closed form approximation formulas for the pricing of simple-asset spread options. However, when the number of asset involved is greater than two, not many approaches are available for computing the spread option price efficiently and accurately, even under the classical Black and Scholes framework. A noticeable work that approximates the multi-asset spread option price is Carmona and Durrleman (2006). While Carmona and Durrleman's method is quite accurate, it suffers from a somewhat major shortcoming in the sense that it does not give the option price in a closed form. To compute the option price, one would have to solve a high-dimensional system of non-linear equation numerically, usually by using the Newton–Raphson's algorithm. However, Deng et al. (2010) indicate that it takes considerable effort to solve them because the convergence of numerical algorithms depends very sensitively on the initial values, and a good understanding of how to choose the initial value is still lacking. Two additional closed form approximations have been recently proposed by Deng et al. (2010) and by Alòs et al. (2011). In particular, the former one extends the approach of Deng et al. (2008), based on a quadratic approximation of the exercise boundary, to the multi-asset spread options pricing

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problem, whereas the latter one extends the approach of Kirk (1995) to the three-assets pricing problem.

Numerical examples concerning the pricing and the hedging of spread options written on three underlyings are reported in Deng et al. (2010) and in Alòs et al. (2011). However, the former one treats the particular case where the three volatilities are equal and the time to maturity of the option is very short. Moreover, since the Deng et al. (2010) formula does not provide the gammas and the cross-gammas in closed form, only the deltas of the option have been calculated. If we look at the numerical examples reported in Alòs et al. (2011), the authors focus only on the accuracy of their approximation method for pricing purposes, without an application to the computation of the Greeks. Finally, both numerical examples do not cover the pricing and the hedging of multi-asset spread options in energy markets.

In this paper, a different point of view is adopted. Instead of proposing a new closed form approximation formula for the multi-asset spread options pricing problem, we propose a "smart" way of re-using the option formulas for the simple-asset spread option pricing. If we focus on energy markets, this can be done by applying a moment-matching procedure to the short leg component in a clean spark/dark spread option, which is defined as a particular combination of gas/coal and carbon. The basic idea is that of approximating the actual process of the short leg value by a sufficiently simple stochastic process. The expression "sufficiently simple" should be interpreted as "simple enough to allow for analytic solutions to the pricing problem at hand", see Brigo et al. (2004).

The approximation happens on the basis of a moment matching principle, which can be stated as follows: set the parameters of the approximating short leg process so that as many moments as possible of the actual short leg process are exactly reproduced. With the usual lack of fantasy, the market choice of an approximating process seems to have fallen onto the log-normal one. The distinctive parameters of such a process being only two (the return's average and standard deviation over the time horizon set by the option to price) this moment-matching procedure can only match the first two moments of the original distribution.

The pricing of the option is then performed once the equivalent correlation parameter between the long leg underlying and the proxy short leg has been calculated. The main advantage associated with the moment matching approach proposed in this paper is a reduction of the dimension of the pricing problem: one can indeed continue using all the option formulas available in the literature for simple-asset spread options written on two underlyings. Besides it, the combined use of an option formula for simple-asset spread options and moment matching technique applied to the short leg component provides a good approximation to the Monte Carlo simulation. It is well-known that the Monte Carlo price and Greeks can be considered as the benchmark since no exact formula is available for the pricing and hedging of multi-asset spread options. The accuracy of our method is even comparable to the one provided by using closed form approximation formulas based on three underlyings, as done, for example, in Deng et al. (2010) or in Alòs et al. (2011), where each underlying entering into the short leg component is treated separately.

The rest of the paper is organized as follows. Section 2 presents the general framework under which our spread option pricing results are derived. In Section 3 we present a first approximation method, namely a two-moment matching procedure to the short leg component including the strike price, from which a Generalized Margrabe (1978) formula for pricing and hedging multi-asset spread options can be derived. In Section 4 a second approximation method is presented, namely a two-moment matching procedure to the short leg component excluding the strike price. This allows us to introduce a Generalized Kirk (1995) formula for pricing and hedging multi-asset spread options. A comparison between the two approaches is reported in Section 5. Due to the richness of results concerning the pricing and hedging of two-legged spread options, which have improved the approximation provided by Kirk (1995), in Section 6 we derive other generalized formulas for pricing and hedging multi-asset spread options, namely the Generalized Ravindran (1993) formula and the Generalized Bjerksund and Stensland (2011) formula. This follows without any additional effort from the second approximation method where the two-moment matching approximation is applied to the short leg component, strike excluded. In Section 7 we discuss the accuracy of the approximation method, by providing with some insights concerning the robustness and possible limitations. In Section 8 we compare our approach with a Monte Carlo simulation and other closed form approximations based on three underlyings for pricing and hedging virtual clean spark power plants. In Section 9 we compare our approach with a Monte Carlo simulation and other closed form approximations based on three underlyings for pricing and hedging virtual clean dark power plants with quanto effect. Section 10 concludes the paper.

2. Assumptions and notation

In what follows we will consider a multi-asset spread option and we will assume a log-normal model for the forward prices for each underlying. The evolution of the forward prices for the $n + 1$ assets is described, under the risk-neutral probability measure $\mathbb{Q}$, by the following stochastic differential equations (SDEs):

$$\frac{dF_i(t,T)}{F_i(t,T)} = \sigma_i(t,T)dW_i(t), \tag{1}$$

for $i \in \{0, \ldots, n\}$, where $F_0(t, T)$, for a fixed T, represents the forward price at time t with maturity T for the long underlying, whereas $F_i(t, T)$, for $i \in \{1, \ldots, n\}$, denote the forward prices at time t with maturity T for the short underlyings. The volatility term structure is described by the deterministic function $\sigma_i(t, T)$, for $i \in \{0, \ldots, n\}$ (see Appendix A for a special choice of the function $\sigma_i(t, T)$).

The $n + 1$ Brownian motions $W_i(t)$, $i \in \{0, \ldots, n\}$, are supposed to be correlated under the risk-neutral probability measure $\mathbb{Q}$, i.e.

$$\mathrm{Corr}\Big(dW_i(t), dW_j(t)\Big) = \rho_{ij}(t)dt,$$

for $i, j \in \{0, \ldots, n\}$, where $\rho_{ij}(t)$ is the correlation coefficient between the underlyings i and j at time t.

Chosen some positive coefficients $\alpha_1, \ldots, \alpha_n \in \mathbb{R}^+$, we are interested in valuing the following multi-asset spread call option with strike price K, whose payoff at maturity T is:

$$C(T,T,K) = \left(F_0(T,T) - \sum_{i=1}^{n} \alpha_i \; F_i(T,T) - K\right)^+, \tag{2}$$

where, as usual, $x^+ = \max\{x, 0\}$. Risk-neutral valuation gives the price $C(T, T, K)$ at time t for a multi-asset spread call option with maturity T and strike price K as the following expectation

$$C(t,T,K) = e^{-r(T-t)}\mathbb{E}[C(T,T,K)|\mathcal{F}_t],$$

under the risk-neutral measure $\mathbb{Q}$, conditional on the information available at time t, where the risk-free rate is supposed to be constant.

3. A Generalized Margrabe formula for pricing and hedging multi-asset spread options

The basic idea is that of approximating the actual process of the short leg component including the strike price, namely

$$F_{\mathrm{SL}}(t,T) \equiv \sum_{i=1}^{n} \alpha_i F_i(t,T) + K, \tag{3}$$

by a sufficiently simple stochastic process $\overline{F}_{\mathrm{SL}}(t,T)$, where the subscript SL stands for short leg.

The expression sufficiently simple should be interpreted as simple enough to allow for analytic solutions to the pricing problem at hand,