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Expected commodity returns and pricing models[☆]

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ABSTRACT

Stochastic models of commodity prices have evolved considerably in terms of their structure and the number and interpretation of the state variables that model the underlying risk. Using multiple factors, different specifications and modern estimation techniques, these models have gained wide acceptance because of their success in accurately fitting the observed commodity futures' term structures and their dynamics. It is not well emphasized however that these models, in addition to providing the risk neutral distribution of future spot prices, also provide their true distribution. While the parameters of the risk neutral distribution are estimated more precisely and are usually statistically significant, some of the parameters of the true distribution are typically measured with large errors and are statistically insignificant. In this paper we argue that to increase the reliability of commodity pricing models, and therefore their use by practitioners, some of their parameters — in particular the risk premium parameters — should be obtained from other sources and we show that this can be done without losing any precision in the pricing of futures contracts. We show how the risk premium parameters can be obtained from estimations of expected futures returns and provide alternative procedures for estimating these expected futures returns.

1. Introduction

Stochastic models of commodity prices have evolved considerably during recent years in terms of their structure and the number and interpretation of the state variables that model the underlying risk (Gibson and Schwartz, 1990; Schwartz, 1997; Schwartz and Smith, 2000; Cortazar and Schwartz, 2003; Cortazar and Naranjo, 2006). Using multiple factors, different specifications and modern estimation techniques, these models have gained wide acceptance because of their success in accurately fitting the observed commodity futures' term structures and their dynamics.

Most of the commodity price models are calibrated using only futures panel data.[1] They assume that there are no-arbitrage opportunities in trading within these contracts and that the underlying process for commodity prices may be derived using only futures prices. These models provide the risk adjusted distribution of future spot commodity prices that, under the risk neutral framework, may be used to price all types of commodity derivatives and real options.

It is not well emphasized however that these models, in addition to providing the risk neutral distribution of future spot prices, also provide their true distribution. Even though the commodity price distribution under the (true) physical measure is unnecessary for valuation purposes, it is still important for at least two reasons. First, the true distribution is useful for non-valuation purposes, such as risk management (i.e. calculations of *Value at Risk*). Second, many practitioners still do not use the risk neutral approach for valuing natural resource investments, but instead use commodity price forecasts and then discount the expected cash flows generated with those forecasts at the weighted average cost of capital.[2]

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[1] Some commodity models use also additional information, including Schwartz (1997) and Casassus and Collin-Dufresne (2005), which consider bond prices and Geman and Nguyen (2005) that incorporate inventory data. Also Cortazar et al. (2008) and Cortazar and Eterovic (2010) formulate multi-commodity models which use prices from one commodity to estimate the dynamics of another, and Trolle and Schwartz (2009) use commodity option prices to calibrate an unspanned stochastic volatility model.

[2] The International Valuation Standards Council (IVSC) released the discussion paper *Valuation in the Extractive Industries* in July 2012. Different questions about valuation methodologies where stated in this paper which industry participants were invited to answer. These answers where published and can be accessed at http://www.ivsc.org/comments/extractive-industries-discussion-paper. Respondents include the Valuation Standards Committee of the SME, The VALMIN Committee, the CIMVal committee and the American Appraisal Associates among others. Most of the respondents stated that their main method of valuation was a discounted cash flow analysis (DCF) using various methods of price forecasting. For the discount factor the most widely used method was a weighted average cost of capital (WACC) based on the Capital Asset Pricing Model (CAPM).

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Thus, not only the risk adjusted process for valuing derivatives is of interest for users of commodity models, but also expected spot prices and their dynamics under the physical measure.

It is well known that expected future spot and futures prices differ only on the risk premiums, since futures prices are expected spot prices under the risk neutral measure. And here lies the problem: while the parameters of the risk neutral distribution are estimated more precisely and are usually statistically significant, some of the parameters of the true distribution are typically measured with large errors and are statistically insignificant (Schwartz, 1997; Cortazar and Naranjo, 2006). Thus, if these risk premiums are not well estimated, even though futures prices may not be affected, expected spot prices under the physical (true) measure will be.[3] So, when these models are used to infer anything about the true distribution of spot prices (e.g. NPV or risk management) they become very unreliable.

In this paper we argue that to increase the reliability of commodity pricing models, and therefore their use by practitioners, some of their parameters — in particular the risk premium parameters — should be obtained from other sources. Using the Schwartz and Smith (2000) model we show that this can be done without losing any precision in the pricing of futures contracts. We show how the risk premium parameters can be obtained from estimations of expected futures returns and provide alternative procedures for estimating these expected futures returns.

The remaining of the paper is as follows: Section 2 illustrates the nature of the problem using the Schwartz and Smith (2000) commodity pricing model, and Section 3 shows how to estimate expected futures returns in this model. Section 4 describes alternative ways of estimating expected future returns and Section 5 presents empirical results of implementing our methodology for copper and oil futures. Finally, Section 6 concludes.

2. An example

To illustrate more precisely the nature of the problem we use the two-factor Schwartz and Smith (2000) commodity model which has been widely used by academics and practitioners.[4]

The first state variable of this model (ξ_t), represents the long term equilibrium (log) price level, while the second state variable (χ_t), represents short term mean-reverting variations in (log) prices. The log spot price (S_t) is then defined in Eq. (1) as the sum of the state variables. Eqs. (2) and (3) present the stochastic processes (under the physical measure) followed by the state variables, where μ_ξ, κ, σ_ξ and σ_χ are parameters of the model.

$$\ln(S_t) = \chi_t + \xi_t \tag{1}$$

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi \tag{2}$$

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \tag{3}$$

Furthermore, dz_ξ and dz_χ are correlated Brownian motions with correlation $\rho_{\chi\xi}$, such that:

$$dz_\chi dz_\xi = \rho_{\chi\xi} dt. \tag{4}$$

Eqs. (5) to (7) present the stochastic processes followed by the state variables under the risk neutral measure, where λ_χ and λ_ξ are the risk premiums which are assumed to be constant.

$$d\xi_t = \left(\mu_\xi - \lambda_\xi\right)dt + \sigma_\xi dz_\xi^Q \tag{5}$$

$$d\chi_t = \left(-\kappa\chi_t - \lambda_\chi\right)dt + \sigma_\chi dz_\chi^Q \tag{6}$$

$$dz_\chi^Q dz_\xi^Q = \rho_{\chi\xi} dt \tag{7}$$

Some relevant results of the Schwartz and Smith (2000) model are the expected value at time t of the state variables at time T, their covariance matrix and the expected value of the spot price. These are presented in Eqs. (8) through (10), respectively.

$$E_t\left(\begin{bmatrix}\chi_T\\ \xi_T\end{bmatrix}\right) = \begin{bmatrix} e^{-\kappa(T-t)}\chi_t \\ \xi_t + \mu_\xi(T-t)\end{bmatrix} \tag{8}$$

$$Cov_t\left(\begin{bmatrix}\chi_T\\ \xi_T\end{bmatrix}\right) = \begin{bmatrix} \left(1-e^{-2\kappa(T-t)}\right)\dfrac{\sigma_\chi^2}{2\kappa} & \left(1-e^{-\kappa(T-t)}\right)\dfrac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} \\ \left(1-e^{-\kappa(T-t)}\right)\dfrac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} & \sigma_\xi^2(T-t)\end{bmatrix}$$

$$E_t(S_T) = \exp\left[e^{-\kappa(T-t)}\chi_t + \xi_t + A'(T-t)\right] \tag{9}$$

$$A'(T-t) = \mu_\xi(T-t) + \frac{1}{2}\left(\left(1-e^{-2\kappa(T-t)}\right)\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2(T-t) + 2\left(1-e^{-\kappa(T-t)}\right)\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right) \tag{10}$$

Furthermore, the price of a futures contract at time t that matures at time T ($F_{T,t}$) is given by the expected spot price under the risk neutral measure ($E_t^Q[S_T]$). This implies that the futures price is:

$$F_{T,t} = \exp\left[e^{-\kappa(T-t)}\chi_t + \xi_t + A(T-t)\right] \tag{11}$$

$$A(T-t) = \left(\mu_\xi - \lambda_\xi\right)(T-t) - \left(1-e^{-\kappa(T-t)}\right)\frac{\lambda_\chi}{\kappa} + \frac{1}{2}\left(\left(1-e^{-2\kappa(T-t)}\right)\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2(T-t) + 2\left(1-e^{-\kappa(T-t)}\right)\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right).$$

Notice that the only difference between Eqs. (10) and (11) are the risk premium parameters (lambdas). If the risk premiums were zero, then futures prices would coincide with expected spot prices.

Consider now an extreme example of the issue we want to illustrate. Between January 2009 and December 2012 COMEX copper prices increased by almost 160% (from 1.40 to 3.65 US$ per pound). Table 1 presents the model parameters estimated using a Kalman[5] filter using all futures price data from this period.

Table 1
Model parameters estimated from copper futures prices, standard deviation (S.D) and t-test. 2009–2012.

Parameter	Kalman filter parameters		
	Estimate	S.D	t-Test
κ	0.111	0.012	9.513
σ_χ	0.910	0.069	13.180
λ_χ	0.036	0.096	0.369
μ_ξ	0.266	0.145	1.833
σ_ξ	0.605	0.143	4.240
μ_ξ^Q	−0.043	0.056	−0.764
$\rho_{\chi,\xi}$	−0.903	0.048	−18.905

Note that instead of estimating μ_ξ and λ_ξ, we follow Schwartz and Smith (2000) and estimate μ_ξ and μ_ξ^Q with $\mu_\xi = \mu_\xi^Q + \lambda_\xi$, which is equivalent. Thus, the expected return restrictions imposed on λ_ξ are actually reflected in the values of μ_ξ.

[3] In an independent work, Heath (2013) also finds that a futures panel is well suited for estimating the cost of carry, relevant for futures prices, but not the risk premiums, required for expected spot prices.

[4] We came across this problem in conversations with a very large mining company which was using this model to value their real options.

[5] More details about the estimation procedure will be presented later in the paper.