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Piecewise smooth approximation of bottom–up abatement cost curves

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ABSTRACT

Top–down models usually include piecewise-smooth functions to describe marginal cost curves, while bottom–up models use step function curves. When a bottom–up cost curve is available, we can explicitly represent this curve with a top–down model in order to replicate its shape instead of using arbitrary assumptions. We propose several methods to approximate a piecewise function from a step function using constant elasticity of substitution technologies. Specifically, we consider a pollution abatement sector and calibrate the parameters of the abatement function in order to allow proper assessment of the economic effects of an environmental policy. Our methodology can be applied to any sector characterized by decreasing returns to scale technologies. We conclude that the elasticities of substitution need not be estimated only on the basis of historical data, but can be precisely calibrated on the basis of engineering estimates of technology potential.

1. Introduction

There is a conventional notion that elasticities of substitution are always estimated on the basis of historical data. They are critical parameters in top–down modeling and provide good approximation of technology options. When elasticities are estimated from historical data, there is no guarantee that the parameter values will remain valid in the future under different abatement policies (Jaccard et al., 2004). We propose a methodology to determine the elasticity of substitution on the basis of engineering studies. Instead of an econometric estimation, we calibrate a bottom–up cost curve to specify the elasticity value.

Top–down models usually include piecewise-smooth functions to describe marginal cost curves, while bottom–up models describe those curves with a step function. When bottom–up cost curve is available, we can explicitly represent this curve with a top–down model in order to replicate its shape instead of using arbitrary assumptions. However, there is a lack of information about the range of alternative activities to which the producer can switch, implying that elasticity of substitution must be assumed. Judgments about the scope of substitution possibilities are discussed in Sue Wing (2006a) and Baker et al. (2008). We show how to identify the elasticity of substitution[1] with bottom–up data. The piecewise-smooth approximation method is explained using a pollution abatement sector, but our methodology can be applied to any sector characterized by decreasing returns to scale technologies.

Relatively few top–down models explicitly specify the production function of pollution abatement activities. Initially Jorgenson and Wilcoxen (1990) assumed that an industry's production function for pollution abatement directly mirrors the production function for its good output. Later Nordhaus and Yang (1996) implemented a quadratic abatement cost curve and calibrated the intercepts of the estimated marginal abatement cost (MAC) curve. Ellerman and Decaux (1998) fitted simple analytical curves to a set of MAC curves and investigated the robustness of MACs with respect to abatement levels among regions. Hyman et al. (2003) implemented a constant elasticity of substitution (CES) abatement function. The authors chose a value of elasticity of substitution and compared it to the bottom–up MAC to allow for an arbitrary adjustment of the fit. Gerlagh et al. (2002) proposed an ordinary least square estimation to cover as much information as possible on the technical measures underlying the abatement options. Boehringer et al. (2006) used an activity analysis to directly incorporate bottom–up function into a top–down model. Revesz and Balabanov (2007) defined an average abatement cost function using a degree of abatement possibilities and a scaling factor. The GEM-E3 model for the European Union (Capros et al., 2008) explicitly specifies MAC as an isoelastic exponential function where installations of abatement technologies are treated as an input rather than an investment.

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[1] It is not the only one parameter that is usually assumed arbitrary in top–down models. An investigation of other parameters goes beyond the scope of that paper.

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In this paper we show an algorithm for a smooth representation of bottom–up cost curves which enables us to portray the isoquant defined by the activity analysis formulation. The benchmark equilibrium describes prices and quantities at a reference point. Properly calibrated, this point will be the same in both the smooth and the step curves. What functional form should be considered? Yu (2005) proposes to capture an abatement activity similar to the iceberg cost together with the standard constant returns to scale production function. However, the abatement process is typically characterized by a decreasing returns to scale technology. In equilibrium analysis, a CES-type function is well suited for studying production process and it is also relatively easy to calibrate. We consider a CES function with decreasing returns to scale. The best fit for the CES elasticity will be that which minimizes the weighted deviation from the bottom–up (engineering) curve.

The inclusion of the bottom–up information on abatement options into a top–down model in the traditional way[2] involves (i) piecewise-smooth approximation that best describes the bottom–up cost curve, (ii) integration of the approximation into a top–down model (Kiuila and Peszko, 2006). We are not going to analyze the methods of including abatement function in top–down models, but we show how to evaluate the parameters of the abatement function to be used in top–down models. We discuss and compare the four methods assuming a decreasing returns to scale technology. A rational polluter, when faced with the necessity to reduce pollution, will utilize the cheapest options first and then turn to more costly ones. The marginal cost curve will therefore be non-decreasing. In addition, a complete emission reduction is not possible via technical measures and a reduction of economic activity is required in order to achieve that goal. Thus the cost curve approaches a vertical asymptote, while the marginal cost approaches infinity.

We discuss the importance of analyzing marginal, rather than total or average abatement cost. We consider a choice between the three cost curves to verify that targeted cost matters during the approximation procedure. We verify this hypothesis using abatement cost curves for greenhouse gasses in the Czech Republic, Poland and Switzerland, estimated by McKinsey & Company. The results for all three curves suggests that it does not matter whether we target marginal or total cost, but the choice between the two and the average cost might matter. We analyze in the paper the details of this experiment for Switzerland and results for other countries are presented in Supplementary material 3.

Finally, we address the issue of negative bottom–up cost. A McKinsey type cost curve gives an illusion that part of pollution abatement can be achieved for free. The construction of the cost curve implies that each action is independent from every other action and the probability of adopting is the same for all new technologies. A wide discussion of the free lunch problem can be found in Holmes (2010). We correct these negative costs using rescaling and compare three approaches, because the results of top–down models are sensitive in this respect. Below-zero costs prove to be inherently problematic.

The main contributions of the paper are (i) a presentation of algorithms to estimate elasticities of substitution using a forward-looking engineering approach instead of a backward-looking econometric one and (ii) a comparison of alternative methods of approximating smooth cost curves. Many modelers just take arbitrary values for elasticity of substitution because the lack of data does not allow them to provide a good econometric estimation. Using our technique, CGE modelers will be able to calculate elasticity of substitution with engineering data. There are comparable techniques in the literature (Dellink, 2005; Hyman et al., 2003; Jorgenson et al., 2008; Sue Wing, 2006b), but our method has several advantages: it works with any output level (including zero), it works with both sectoral and economy-wide bottom–up data, and it allows for a good match of the bottom–up curve (in a sense that simulations done with smooth and step curves gave similar results).

The rest of the paper is organized as follows. Section 2 explains how to represent a decreasing returns to scale technology with top–down modeling. We show a relationship between the Marshallian concept of supply function and the Arrow–Debreu production function. We complete this section with a discussion of alternative calibration strategies to approximate an abatement curve. The details of the algorithm are available in the Supplementary material 1. In Section 3, we use Swiss data to approximate both the marginal and the total abatement cost curves for greenhouse gasses. Several rescaling methods are applied in order to avoid negative cost. Section 4 concludes.

2. Calibration of marginal cost function

Given the abatement technologies within a bottom–up model, the MAC curve represents the marginal loss in profits from avoiding the last unit of emission given some level of output. In a top–down model, the MAC curve is defined as the shadow cost that is generated by the constraint on pollution emissions. Thus the MAC for a given economy represents the social cost of the last unit of emissions abated. The question is: how should this social cost function be calibrated? We explain this issue using a CES technology. First, the integration of the Marshallian concept into the Arrow–Debreu models is presented. Next, different calibration approaches are explained. The choice of functional form depends on several properties (Ginsburgh and Keyzer, 2002, p. 56; Perroni and Rutherford, 1998; Shoven and Whalley, 1992, p. 94). The CES production function yields convenient analytical expression: the cost function is also CES type. Sato (1976) showed that such expression is available only for a few other forms. Flexible forms, like the translog, do not lead to an explicit form for the cost function under cost minimization.

2.1. Decreasing returns to scale

The marginal abatement cost is nondecreasing, and a strictly convex technology represents the pollution abatement processes where the output changes less than proportionally to inputs. We have to find out a function that can describe such a curve. Let us describe the pollution abatement service Q using a technological potential X and expenditures K, where expenditures include capital, labor, and materials necessary for the abatement process once the abatement technology has been chosen (Fig. 1a). The potential to reduce pollution through technical abatement activities provides an upper bound on the abatement in the model. The remaining part of the pollution can be reduced only by decreasing the economic activity.

When abatement capacity X is in fixed supply, the production function $Q = f(K, \overline{X})$ exhibits decreasing returns to scale in the variable input K (Fig. 1b). The variable input includes capital, labor, and materials necessary for the abatement process. Following Cretegny and Rutherford (2004), there is therefore no loss of generality by formulating the model on the basis of a constant returns to scale CES technology with a fixed factor:

$$Q = \phi\left(\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)\overline{X}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$$

where ϕ is a scale factor, α is a distribution parameter, and σ is a parameter of elasticity of substitution between the abatement capacity and the required expenditures on abatement. It gives a linear expansion path of the cost minimization problem represented in Fig. 1a. In order to abate one unit of emissions, we need an abatement technology and maintenance. Once we have decided which technology to apply, the abatement level will be determined by the input K. The decreasing returns to scale technology implies that the abatement level increases less than proportionally to this input.

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[2] Integration of bottom–up cost with top–down modelling is possible either with a smooth abatement function (traditional approach) or with an activity analysis approach for a step abatement function (hybrid approach).