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The stochastic seasonal behavior of energy commodity convenience yields☆

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ABSTRACT

This paper contributes to the commodity pricing literature by consistently modeling the convenience yield with its empirically observed properties. Specifically, in this paper, we show how a four-factor model for the stochastic behavior of commodity prices, with two long- and short-term factors and two additional seasonal factors, may accommodate some of the most important empirically observed characteristics of commodity convenience yields, such as the mean reversion and stochastic seasonality. Based on this evidence, a theoretical model is presented and estimated to characterize the commodity convenience yield dynamics that are consistent with previous findings. We also show that commodity price seasonality is better estimated through convenience yields than through futures prices.

1. Introduction

In the case of consumption commodities (commodities that are consumption assets rather than investment assets), the benefit from holding the physical asset net of the storage cost is sometimes referred to as the "convenience yield" provided by the commodity (see for example Hull, 2003).

In other words, if we denote by F_t and S_t the futures and spot prices, respectively, in the case of consumption commodities, we do not necessarily have equality in $F_t \le S_t \cdot e^{(r+u)\cdot(T-t)}$ (where r and u represent the risk-free rate and storage costs, respectively, and $T-t$ is the time to maturity) because users of a consumption commodity may feel that ownership of the physical commodity provides benefits that are not obtained by holders of futures contracts. For example, an oil refiner is unlikely to regard a futures contract on crude oil as equivalent to crude oil held in inventory. The crude oil in inventory can be an input to the refining process, whereas a futures contract cannot be used for this purpose. In general, ownership of the physical asset enables a manufacturer to keep a production process running and perhaps profit from temporary local shortages. A futures contract does not do the same (see, for example, Brennan and Schwartz, 1985). Therefore, the convenience yield net of storage costs, denoted by δ^*, is defined such that $F_t \cdot e^{\delta^*\cdot(T-t)} = S_t \cdot e^{r\cdot(T-t)}$.

Previous studies have considered the convenience yield as a deterministic function of time, such as those by Brennan and Schwartz (1985), or as a stochastic process, such as those by Gibson and Schwartz (1990) and Schwartz (1997). Specifically, Gibson and Schwartz (1990) allow for stochastic convenience yield of crude oil to develop a two-factor oil contingent claims price model. Moreover, Gibson and Schwartz (1990) show that convenience yields exhibit mean reversion, which is consistent with the theory of storage (see, for example, Brennan, 1958) in which an inverse relationship is established between the net convenience yield and the inventory level. Schwartz (1997) presents and empirically compares several factor models in which the convenience yield is assumed to be a stochastic factor. Hilliard and Reis (1998) and Miltersen and Schwartz (1998) use models with stochastic convenience yield to value commodity derivatives (futures and options). More recently, Casassus and Collin-Dufresne (2005) characterize a three-factor model, "maximal" in a sense of Dai and Singleton (2000), of commodity spot

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prices, convenience yields and interest rates, which nests many existing specifications.

Wei and Zhu (2006) investigate the empirical properties of convenience yields in the US natural gas market, finding that convenience yields are highly variable and economically significant, with their variability depending on the spot price level, the spot price variability and the variability of lagged convenience yields. Liu and Tang (2011) propose a three-factor model for spot prices, interest rates and convenience yields, accounting for heteroskedasticity in the convenience yield.

Although there have been many papers analyzing the seasonal behavior of some commodity prices (Garcia et al., 2012; Lucia and Schwartz, 2002; Manoliu and Tompaidis, 2002; Sorensen, 2002, among others), considerably less attention has been paid to the seasonal behavior of convenience yields. Based on the finding of seasonality in the convenience yield made by Fama and French (1987), Amin et al. (1995) propose a one-factor model for the spot price with a deterministic seasonal convenience yield. More recently, Borovkova and Geman (2006) present a two-factor model in which the first factor is the average forward price, instead of the commodity spot price, and the second factor is the stochastic convenience yield. These authors allow for a deterministic seasonal premium within the convenience yield.

In this paper, we go further by presenting a three-factor model in which the (stochastic) convenience yield exhibits stochastic seasonality. As we know, the convenience yield is not directly observable. Therefore, we must infer it from futures prices. In a first stage, we estimate the convenience yield following the standard produce suggested by Gibson and Schwartz (1990), showing that the estimated convenience yield exhibits mean-reversion and (stochastic) seasonality. In a second stage, we provide another estimation for the convenience yield based on the four-factor model by Garcia et al. (2012). Specifically, we show that the four-factor model presented by Garcia et al. (2012), with two long- and short-term factors and two additional trigonometric seasonal factors, may generate stochastic seasonal convenience yields. An expression for the instantaneous convenience yield within this model is obtained, showing that the instantaneous convenience yield exhibits mean reversion and stochastic seasonality. Moreover, a π/2 lag is found in the convenience yield seasonality with respect to spot price seasonality.

Based on this evidence, the next step is to present a theoretical three-factor model to characterize the commodity convenience yield dynamics consistently with previous findings, which is the main contribution of the paper. Specifically, the model considers mean reversion and stochastic seasonal effects in the convenience yield. The model is estimated using data from a variety of energy commodity futures prices: crude oil, heating oil, gasoline and natural gas. We also show that commodity price seasonality is better estimated through convenience yields rather than through futures prices. The reason is that futures prices are driven for many things, such as supply, demand, political aspects, speculation, weather conditions, etc. Therefore, it may sometimes be difficult to extract the seasonal component from futures prices. However, as we will show in Section 2, the convenience yield is estimated through a ratio of two futures prices, so many of these non-seasonal factors tend to disappear, facilitating the estimation of the seasonal component.

Finally, the three-factor model for the convenience yield is not only useful to help in the understanding of the stochastic behavior of the convenience yield, but it is also useful for commodity derivative valuation purposes. Specifically, we show that it is better to obtain estimations for futures prices in those cases in which there are missing data by means of the three-factor model for the convenience yield, rather than by means of the four-factor model for futures prices. Furthermore, the model for the convenience yield is simpler and consequently easier to estimate than the model for futures prices.

The remainder of this paper is organized as follows. Section 2 presents the data and some preliminary findings regarding seasonality in convenience yields. We show that convenience yields exhibit mean reversion and stochastic seasonality, using data from crude oil, heating oil, gasoline and natural gas futures markets. In Section 3, we present the four-factor model accounting for stochastic seasonality in commodities and the expression for the instantaneous convenience yield derived from this four-factor model. In Section 3, we also discuss the properties of the model-estimated convenience yields for the four commodities under study, showing that they in fact exhibit mean reversion, stochastic seasonality and a π/2 lag with respect to spot price seasonality. Based on this empirical evidence, in Section 4, a three-factor model is proposed and estimated characterizing the commodity convenience yield dynamics, considering mean reversion and stochastic seasonal effects in the convenience yield. In Section 5 we present some applications of the three-factor model for the convenience yield to commodity derivative valuation. Finally, Section 6 concludes with a summary and discussion.

2. Data and preliminary findings

In this section, we present a data description of the futures prices for the four commodities used in the paper, i.e., WTI crude oil, heating oil, RBOB gasoline and Henry Hub natural gas. In addition, the procedure presented by Gibson and Schwartz (1990) is described to obtain the convenience yield data. The section concludes by analyzing the main empirically observed characteristics of the convenience yield data.

2.1. Data description

2.1.1. Futures prices

The data set used in this paper consists of weekly observations of WTI (light sweet) crude oil, heating oil, unleaded gasoline (RBOB) and natural gas futures prices traded on NYMEX during the period 9/27/1999 to 7/4/2011 (615 weekly observations).

Futures are traded on NYMEX with maturities from one month up to seven years for WTI crude oil, from one to eighteen months for heating oil, from one to twelve months for RBOB gasoline and from one month

Table 1

Descriptive statistics. Futures prices. The table shows the mean and volatility of the four commodity futures prices series. The sample period is 9/27/1999 to 7/4/2011 (615 weekly observations). F1 is the futures contract closest to maturity, F2 is the contract second-closest to maturity and so on.

WTI crude oil			Heating oil			Gasoline			Henry Hub		
	Mean	Volatility		Mean	Volatility		Mean	Volatility		Mean	Volatility
F1	55.06	31.30%	F1	64.46	31.73%	F1	64.59	36.81%	F1	5.68	46.80%
F4	55.59	26.49%	F3	64.96	28.08%	F3	64.19	30.13%	F5	6.04	32.53%
F7	55.57	23.83%	F5	65.17	26.04%	F5	63.73	26.26%	F9	6.17	26.91%
F11	55.36	21.69%	F7	65.27	24.08%	F7	63.37	24.53%	F14	6.15	22.48%
F14	55.17	20.57%	F10	65.23	21.59%	F9	63.24	24.31%	F18	6.13	20.80%
F17	54.98	19.72%	F12	65.13	20.61%	F12	63.00	23.77%	F22	6.06	21.55%
F20	54.80	19.05%	F14	65.07	20.10%	–	–	–	F27	5.99	19.57%
F24	54.60	18.44%	F16	65.04	20.04%	–	–	–	F31	5.96	20.05%
F27	54.48	18.13%	F18	65.02	19.95%	–	–	–	F35	5.89	19.17%