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A Mixed Integer Linear Programming model of a zonal electricity market with a dominant producer

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ABSTRACT

We consider a liberalized electricity market, divided in zones interconnected by capacitated transmission links, where a large dimensional power producer operates. We introduce a model for determining the optimal bidding strategies of the large dimensional producer, so as to maximize his own market share, while guaranteeing an annual profit target and satisfying technical constraints. The model determines the optimal medium-term resource scheduling and yields the hourly zonal electricity prices, as it includes constraints representing the Market Clearing process. In order to compute the global solution, the complementarity conditions are formulated as mixed integer linear constraints and the revenue terms are expressed by piece-wise linear functions. The model can be used for analyzing the behavior of market prices in electricity markets where a large dimensional producer can exert market power. It can also be used by investors as a simulation tool for evaluating both the impact on the market and the profitability of investment decisions in the zonal electricity market. A case study related to the Italian electricity market is discussed.

1. Introduction

In some countries the structure of the electric power industry, resulting from the privatization and liberalization process, is such that the former monopolist enjoys significant market power. Regulatory constraints to the incumbent's offer strategy are usually introduced, like an average price-cap, or an absolute price-cap, or a cap on the geographic price-differentiation. Many models of strategic interaction on networks have been developed, see the reviews in Ventosa et al. (2005), Ramos et al. (1998) and Smeers (1997); see also Barquin et al. (2008), Day et al. (2002), Hobbs (2001), Hobbs et al. (2000), Schuler (2001), Li and Tesfatsion (2009) and Somani and Tesfatsion (2008).

In this paper we consider an electricity market divided in zones, interconnected by capacitated transmission links, and characterized by the presence of a dominant producer, who may exert market power to achieve a predetermined annual gross-margin target, whereas the other firms behave like a competitive fringe. Indeed, it is often recognized that incumbents do not take full advantage of their capability to control prices in order to maximize profits. The representation of this assumption on the dominant producer's behavior has the advantage of being easily implementable, since in most markets the dominant company statements to the financial markets report the revenue targets and information about the generating capacity is generally available. Moreover, it is assumed that, among all solutions that guarantee the annual profit target, the dominant producer prefers one that maximizes the annual market share.

In order to solve the dominant producer problem, we propose a two-stage procedure. The first stage is based on a linear programming model that computes, while assuming perfect competition among producers, the hourly zonal prices and the accepted bids determined by the Market Operator, given the hourly zonal demands and the power producers' hourly bids for every generation plant. The second stage is based on the dominant producer's annual resource scheduling model, in which the interdependencies between the hourly zonal prices and the dominant producer's hourly production decisions are expressed by the optimality conditions of the Market Clearing problem. The optimal solution of this model determines the hours in which the dominant producer can increase his profits by modifying the perfect competition solution. The nonlinearities in the Market Clearing optimality conditions are eliminated by using a binary variable formulation of the complementarity conditions; the nonlinear revenue terms in the profit constraint are substituted by piece-wise linear functions. In this way a Mixed Integer Linear Programming formulation of the dominant producer model is obtained, whose solution yields a global optimum. The optimal solution can be efficiently computed by commercial codes, when the problem dimension is not too big. For the cases in which the model dimension becomes a substantial issue, we developed an iterative algorithm, based on a model for the single hour t, that exploits the quasi-separability of the problem with respect to the hours.

The dominant producer model solution yields the hourly zonal electricity prices and therefore can be used by investors as a simulation tool for analyzing both the impact on the market and the profitability

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of investment decisions in the zonal electricity market. Spot market price forecasts are crucial to assessing profitability of investments in electricity generation capacity. Since electricity is not storable and demand is highly inelastic, electricity prices are highly variable in time and the presence of very large incumbents adds considerably to the complexity of price-forecasting. Profitability of marginal generators might depend crucially on high spot prices resulting in a limited number of hours. Further, in some markets, locational price differentiation is implemented in case of transmission congestion, so that profitability of a generator depends on where it is located.

The paper is organized as follows. In Section 2 the model used in the first stage of the procedure is presented, namely the Market Clearing model used by the Market Operator for determining the equilibrium hourly zonal prices and productions. In Section 3 the Mixed Integer Linear Programming model of the dominant producer problem is introduced, on which the second stage of the procedure is based. In Section 4 some tests related to the Italian electricity market are discussed. Future work planned is described in Section 5 and in the Appendix the algorithm is presented for computing the optimal solution of large dimensional problems.

2. The Market Clearing problem

Let Z denote the set of zones in which the network is divided and let L denote the set of transmission links among zones. Let $A_{z,l}$ denote the (z,l)-entry of the network incidence matrix that represents the electricity market, with $A_{z,l}=-1$, if link l leaves from zone z, $A_{z,l}=1$, if link l enters zone z, and $A_{z,l}=0$, otherwise. Let K denote the set of power plants owned by the dominant producer and let $K_z\subseteq K$ denote the set of dominant producer's power plants located in zone z. Analogously, let J denote the set of power plants owned by the competitors and let $J_z\subseteq J$ denote the set of competitors' power plants located in zone z. Let T denote the set of hours contained in the time period. Let $C_{z,t}$ denote the inelastic load demand in zone z in hour t and let $\underline{TR}_{l,t}$ and $\overline{TR}_{l,t}$ denote the minimum and maximum power flow on transmission link l in hour t. Let $\overline{Q}_{k,t}$ denote the quantity offered in the sell bid related to the dominant producer's plant $k\in K$ and let $B_{k,t}$ denote the associated bid price, i.e. the minimum price at which the producer is willing to sell the offered quantity. Analogously, let $\overline{q}_{j,t}$ and $b_{j,t}$ denote quantity and price, respectively, of the sell bid related to the competitors' plant $j\in J$.

In every hour t the Market Operator solves the Market Clearing problem, i.e. for every zone $z\in Z$ the Market Operator chooses the cheapest sell bids in order to satisfy the load $C_{z,t}$ and determines the zonal electricity price $\pi_{z,t}$. The Market Clearing solution yields the accepted quantities $Q_{k,t}$ of the dominant producer's sell bids $k\in K$, the accepted quantities $q_{j,t}$ of the competitors' sell bids $j\in J$ and the power flows $TR_{l,t}$ on the transmission links $l\in L$. The Market Clearing problem in hour t is represented by the Linear Programming model

$$min\sum_{j\in J} b_{j,t}\cdot q_{j,t}+\sum_{k\in K} B_{k,t}\cdot Q_{k,t} \tag{1}$$

subject to

$$\sum_{j\in J_z} q_{j,t}+\sum_{k\in K_z} Q_{k,t}+\sum_{l\in L} A_{z,l}\cdot TR_{l,t}=C_{z,t} \qquad z\in Z \tag{2}$$

$$\underline{TR}_{l,t}\le TR_{l,t}\le\overline{TR}_{l,t} \quad l\in L \tag{3}$$

$$0\le q_{j,t}\le\overline{q}_{j,t} \quad j\in J \tag{4}$$

$$0\le Q_{k,t}\le\overline{Q}_{k,t} \quad k\in K. \tag{5}$$

Constraint (2) requires that in every zone the hourly load demand equals the sum of the hourly productions of all power plants located in the zone, plus the production imported from the connected zones, minus the production exported to the connected zones. Constraint (3) guarantees transmission security, i.e. the power flow on every transmission link is within the lower and upper bounds. Constraint (4) imposes that for every competitors' bid the quantity accepted by the Market Operator does not exceed the offered quantity. Constraint (5) imposes analogous constraint on the quantity accepted by the Market Operator for the dominant producer's bids. The decision variables $q_{j,t}$, $Q_{k,t}$ and $TR_{l,t}$ have to be determined so as to minimize the social cost function (1). The hourly zonal prices are the optimal values of the dual variables $\pi_{z,t}$ associated to the zonal balance constraint (2).

If the dominant producer offers his own production at marginal cost (i.e. he behaves like the competitive fringe, so that $B_{k,t}$, as well as $b_{j,t}$, represent marginal costs), then the Market Clearing model yields the Perfect Competition solution: let $(q^*_{j,t}, Q^*_{k,t}, TR^*_{l,t})$ denote the Market Clearing solution in perfect competition and let $\pi^*_{z,t}$ denote the corresponding hourly zonal prices.

The dominant producer can exert market power and force zonal prices to become higher than in the perfect competition solution. This may happen in two situations:

1. the dominant producer can offer his own production at the price-cap $\overline{\pi}_z$ in zone z when the competitors' capacity in hour t (taking into account the transmission limits) is not sufficient to satisfy the load demand $C_{z,t}$;
2. the dominant producer can reduce his own production (i.e. $Q_{k,t}<Q^*_{k,t}$ for some k), while leaving the bid price unchanged, when the competitors' capacity in hour t is sufficient to satisfy the load demand $C_{z,t}$: this results in more expensive competitors' bids to be accepted by the Market Operator (i.e. $q_{j,t}>q^*_{j,t}$ for some j) and therefore in higher clearing prices.

A profit maximizing behavior would require the dominant producer to exert market power whenever possible, but such a behavior would not be sustainable by the system. Therefore, rather than aiming at profit maximization, the dominant producer aims at obtaining an annual profit target P, compatible with the system, while maximizing the sum of the accepted quantities $Q_{k,t}$ of his own sell bids over all hours in the year, i.e. his own annual market share. The dominant producer problem is then to determine, for every plant and every hour in the year, the quantity he has to bid and the associated bid price, so as to guarantee that the annual profit target is met, while maximizing the annual market share. The dominant producer can be reasonably assumed to be able to estimate competitors' bids and transmission limits (indeed in Italy, for instance, the dominant producer is the former monopolist). Therefore, by means of the Market Clearing model, the dominant producer can determine the perfect competition solution (i.e. the Market Clearing solution that the Market Operator would obtain if the dominant producer offered his own production at marginal cost). The dominant producer then needs a model for determining how to modify the perfect competition solution in order to meet the yearly profit target: such a model, introduced in the following section, yields the bid quantities and the bid prices for all dominant producer's power plants in all hours of the year, so as to guarantee the annual profit target and the maximization of the annual market share.

3. A Mixed Integer Linear Programming model for the dominant producer

In this section we introduce a Mixed Integer Linear Programming model for determining how the dominant producer has to modify the perfect competition solution, in order to obtain the annual profit target.

The model decision variables are $q_{j,t}$, $Q_{k,t}$, $TR_{l,t}$, $\pi_{z,t}$, $\eta^-_{l,t}$, $\eta^+_{l,t}$, $\lambda^-_{j,t}$ and $\lambda^+_{j,t}$, for $j\in J$, $k\in K$, $l\in L$, $z\in Z$ and $t\in T$. Variables $q_{j,t}$, $Q_{k,t}$ and $TR_{l,t}$ are