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Revisiting the evidence for cardinal treatment of ordinal variables

Carsten Schröder[a,*], Shlomo Yitzhaki[b]

[a] DIW Berlin/SOEP and School of Business & Economics, Free University Berlin, Germany
[b] Hebrew University and Hadassah Academic College, Israel

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ABSTRACT

Well-being (life satisfaction or happiness) is a latent variable that is impossible to observe directly. Moreover, it does not have a unit of measurement. Hence, survey questionnaires usually ask people to rate their well-being in different domains. The common practice of comparing well-being by means of averages or linear regressions ignores the fact that well-being is an ordinal variable. Since data is ordinal, monotonic increasing transformations are permissible. We illustrate the sensitivity of empirical studies to monotonic transformations using examples that relate to well-known empirical papers, and provide two theoretical conditions that enable us to rank ordinal variables. In our examples, monotonic increasing transformations can in fact reverse the conclusion reached.

1. Introduction

There is enormous interest in the social sciences in the relationships between psychological well-being (life satisfaction or happiness) and socioeconomic outcomes. Several initiatives, the most influential being the Commission sur la Measure de la Performance Économique et du Progrès Social, suggest that economic performance measures (such as GDP) be used together with well-being indicators (Stiglitz et al., 2009, 2010). Since as early as 1972, Bhutan has been measuring its "gross national happiness," and several other countries (Thailand, Australia, China, France, and the United Kingdom) are currently developing comparable indices. White (2007) proposed the Satisfaction with Life Index in an attempt to provide aggregate happiness statistics that are comparable across nations.

Well-being (life satisfaction or happiness) is a latent variable that is impossible to observe directly and that has no natural quantitative measurement unit. Data are collected in scientific surveys that ask questions like, "All in all, how satisfied are you with your life at the moment?" Respondents answer these questions by ranking their satisfaction levels on a pre-defined scale, usually ranging from 4 to 11 points (and sometimes more), with the individual points assigned terms such as "very bad," "bad," or "good."[1] In a scale like this, we know that "very bad" is lower than "bad" and that "bad" is lower than "good", but it is not clear whether the distance between "very bad" and "bad" is greater or smaller than the difference between "bad" and "good". The ordinal nature of this data means that any monotonic increasing transformation of the scale is allowed.

There are various econometric techniques, including ordered probit and logit models or nonparametric methods, that allow for

* Corresponding author.
E-mail addresses: cschroeder@diw.de (C. Schröder), shlomo.yitzhaki@mail.huji.ac.il (S. Yitzhaki).

[1] For interdisciplinary conventions to operationalization of well-being, see Diener (2006).

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empirical analysis of ordinal data. Still, many empirical investigations rely on techniques designed for quantitative cardinal variables.[2] For example, Easterlin *et. al.* (2012) compare averages of different life satisfaction measures showing that, "China has followed the life satisfaction trajectory of the central and eastern European transition countries—a U-shaped swing and a nil or declining trend" [p. 9775]. Dunn and Norton (2012) interpret the well-being scale as a ratio scale and argue that, "Our data showed that people who earned $55,000 were just 9% more satisfied than those making $25,000."

The application of statistical methods designed for quantitative data to ordinal data (comparisons of means or OLS regressions) may be justified by a more intuitive interpretation of results, or for the purposes of descriptive analysis or the benchmark analysis in experimental studies. However, it is crucial to ensure that the application does not lead to biased results: That is, the empirical findings should be robust to monotonic increasing transformations of the well-being scale. The empirical findings of Ferrer-i-Carbonell and Frijters (2004) are frequently cited to justify the use of quantitative techniques for investigating empirical relationships in well-being data. The OECD (2013) made reference to this paper when arguing that treating ordinal data "as cardinal does not generally bias the results obtained (Ferrer-i-Carbonell and Frijters, 2004)" (OECD, 2013, p. 174). However, the analysis of Ferrer-i-Carbonell and Frijters (2004) is not sufficient to justify the OECD's conclusion. This is because the authors investigate the robustness of empirical relationships *from a given* well-being scale (0–10) to selected econometric models (e.g., OLS vs ordered logit, random vs. fixed effects, etc.). They do not investigate the robustness of the results *to monotonic increasing transformations* of the well-being scale.

The present paper makes two contributions to the literature. In the theoretical part, we show two conditions under which the use of methods intended for quantitative cardinal variables in the context of ordinal variables produces results that are robust to monotonic increasing transformations of the ordinal variable.[3] These two conditions are a valuable tool for empirical researchers because they reveal whether there exist monotonic increasing transformations of the ordinal variable that can reverse the empirical results. If the result can be reversed, estimation methods for cardinal variables will also yield biased estimates (the magnitude of the coefficients is affected). Hence, our investigations can be understood as concrete counterexamples showing that estimation methods for cardinal data are not appropriate.[4]

The first condition is relevant for assessing the robustness of comparisons of means. The second condition is relevant for the robustness of the sign of the simple OLS regression coefficient. The term "simple" refers to a regression with one explanatory variable. The first condition is well-known from expected utility theory, the economics of inequality, and studies on educational achievement (see, e.g., Hanoch and Levy (1969), Hadar and Russell (1969), and Spencer (1983)): Under first-order stochastic dominance of one distribution compared to another, rankings of expected values are invariant to monotonic increasing transformations. Otherwise, it would be possible to find monotonic increasing transformations that would change the rankings.

The second condition relies on the concept of the so-called LMA curve (line of independence minus absolute concentration curve; for details, see Yitzhaki and Schechtman, 2012, 2013; Appendix A2 provides an example for the construction of an LMA curve). The LMA curve is the vertical difference between two curves:

1. The first curve is the absolute concentration curve of a variable Y given X, under the assumption that the two variables are statistically independent. Because of statistical independence, this curve is a line (L).
2. The second curve is the *A*bsolute concentration curve of Y as a function of the cumulative distribution $F(X)$.

If the LMA does not intersect the horizontal axis, it is impossible to change the sign of the simple OLS regression coefficient by means of a monotonic increasing transformation. Otherwise, transformations may affect the sign by enlarging or shrinking areas between the LMA curve and the horizontal axis.

Our findings point to the superiority of Gini's mean difference (see Yitzhaki and Schechtman (2013)) over OLS regressions, if the main purpose of the study is to deal with ordinal variables.[5] In the review of the methodology, we point out additional properties of the superiority of Gini's mean difference over the variance, in cases that the distribution of the explanatory variables is not multivariate normal.

Our conditions are not only relevant in the area of happiness, satisfaction, and well-being, but in all settings that treat ordinal variables as cardinal variables in order to simplify the empirical work. One important setting is the measurement of educational achievement and evaluation of education programs based on exam scores compiled by the OECD Programme for International Student Assessment (PISA).[6] This is because, "as with utility functions, any monotonic increasing transformation of the test score scale is also potentially a valid scale" (Bond and Lang, 2013, p. 1468).[7] Other settings include research on economic status or social class (low, middle, or high), health outcomes, and positions in competitions (first, second, third).

[2] For possible reasons, see Section 5.3.

[3] We thank an anonymous referee for making this point. We assume that the monotonic increasing transformation is identical for all individuals. If different groups of people have different perceptions of the described ordinal scale, one could use dummy variables to "control" for heterogeneity of perceptions across groups. Oswald and Wu (2010) analyze the meaningfulness of individuals' subjective well-being assessments with an objective measure—spatial compensating differentials—and find a strong state-by-state match between subjective and objective well-being.

[4] We thank an anonymous referee for making this point.

[5] A STATA procedure to estimate the GMD regression and LMA curves is written by Schaffer (2015).

[6] See Schröder and Yitzhaki (2016b) on this issue.

[7] In their empirical analysis of the sensitivity of student scores to order-preserving transformations, Bond and Lang (2013) conclude: "while many findings will be robust to scale changes, many will not be" (p. 1479). Spencer (1983) develops several propositions concerning the treatment of ordinal variables in the area of education, but these appear to have gone unnoticed in the literature to date.