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Heterogeneous risk/loss aversion in complete information all-pay auctions

Zhuoqiong (Charlie) Chen[a,b], David Ong[c,*], Ella Segev[d]

[a] Harbin Institute of Technology, Shenzhen, University Town, Nanshan District, 518055 Shenzhen, China
[b] London School of Economics, Houghton Street, WC2A 2AE London United Kingdom
[c] Peking University, HSBC Business School, University Town, Nanshan District, 518055 Shenzhen, China
[d] Ben Gurion University of the Negev, P.O.B 653, 84105 Beer Sheva, Israel

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ABSTRACT

We extend previous theoretical work on n-player complete information all-pay auctions to incorporate heterogeneous risk- and loss-averse utility functions. We provide sufficient and necessary conditions for the existence of equilibria with a given set of active players with any strictly increasing utility functions and characterize the players' equilibrium mixed strategies. Assuming that players can be ordered by their risk aversion (player a is more risk-averse than player b, if whenever player b prefers a certain payment over a given lottery, so does player a), we find that in equilibrium, the more risk-averse players either bid higher than the less risk-averse players and win with higher ex-ante probability – or they drop out. Furthermore, while each player's expected bid decreases with the other players' risk aversion, her expected bid increases with her own risk aversion. Thus, increasing a player's risk aversion creates two opposing effects on total expected bid. A sufficient condition for the total expected bid to decrease with a player's risk aversion is that this player is relatively more risk-averse compared to the rest of the players. Our findings have important implications for the literature on gender differences in competitiveness and for gender diversity in firms that use personnel contests for promotions.

1. Introduction

Sunk cost contests, where effort is unrecoverable, are pervasive (Frank and Cook, 2010). All-pay auctions are those where the winners need only exert slightly higher effort to take all. Indeed, all-pay auctions theory has been used to study many types of sunk cost contest and tournaments, e.g., rent seeking contest and lobbying (Baye et al., 1993; Ellingsen, 1991; Hillman and Riley, 1989), election campaigns (Che and Gale, 1998), R&D races (Dasgupta et al., 1982), curved grades (Andreoni and Brownback, 2014), college admission (Hickman, 2014), and job promotion (Rosen, 1986). In these contests, the risk of lost effort, opportunities, or resources to individuals can be significant. Furthermore, even contests between organizations, like firms, can involve significant loss to individuals to the extent that decisions are made by CEOs and managers who care about the consequences of those decisions on their own welfare through mechanisms such as options in compensation packages (Bertrand, 2009), and of course, in promotions and dismissals based upon relative performance. Heterogeneous risk aversion (e.g., as indicated by gender) could thus have a significant influence on behavior.

* Corresponding author.
E-mail addresses: chenzq926@gmail.com (Z. (Charlie) Chen), dvdong@gmail.com (D. Ong), ellasgv@bgu.ac.il (E. Segev).

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Evidence is accumulating of a gender difference in risk aversion, where women are usually found to be more risk-averse than men (Borghans et al., 2009; Charness and Gneezy, 2012; Croson and Gneezy, 2009). This gender difference emerges even before adolescence (Khachatryan et al., 2015). A gender difference in risk attitude and its interactions with all-pay auction incentives in the business world can help explain the paucity of women among top executives (Bertrand, 2009), particularly in entrepreneurial settings (Coates et al., 2009). However, despite the importance of observable differences in attitude towards risk in such contests, the modeling of all-pay auction incentives has generally been limited to risk neutral players or to specific tractable utility functional forms.

In order to fill the gap in the theory of all-pay auctions, we extend Baye, Kovenock, and De Vries's (1996) n-player, complete information all-pay auction model to incorporate heterogeneous risk-averse players. We provide sufficient and necessary conditions for an equilibrium with a given set of active players to exist and more importantly, closed-form solutions to the equilibrium strategies for any strictly increasing utility functions. Assuming that players can be ordered by their risk aversions (player a is more risk-averse than player b, if whenever player b prefers a certain payment over a given lottery, so does player a), we derive novel comparative statics for equilibria in which active players randomize continuously from 0 to the common value of the prize.

We find that, in equilibrium, the more risk-averse players either bid higher than the less risk-averse players (in terms of first-order stochastic dominance of their mixed strategy cumulative distribution) and win with higher ex-ante probability – or they drop out. When players are homogeneous in their risk aversion, the total expected bid decreases with their risk aversion. We also find, in the heterogeneous risk aversion case, that while each player's expected bid decreases with the other players' risk aversion, her expected bid increases with her own risk aversion. Thus, increasing a player's risk aversion creates two opposing effects on total expected bid. A sufficient condition for the total expected bid to decrease with a player's risk aversion is that this player is relatively more risk-averse compared to the rest of the players.

Our findings have important implications for the gender differences in competitiveness literature. With only two risk aversion types of players, e.g., men and women, we show that the total expected bid decreases monotonically with the share of the more risk-averse players, when the difference between the two types is not too large. Moreover, our findings suggest the possibility that if women are more risk-averse than men, they can simultaneously work harder than men and decrease everyone's effort in the firm in personnel contests that have an all-pay auction structure. In these contests, if men and women are not too different in their levels of risk aversions, then a higher share of women may lead to increased odds of a specific woman dropping out. We discuss the specific results in the gender differences in competitiveness literature that these findings can help explain after the main results.

1.1. Literature review

As is always the case with equilibria of complete information all-pay auctions with more than two players, there is no uniqueness of the equilibrium. In fact, there is a continuum of equilibria as in Barut and Kovenock (1998), Baye et al. (1996), Siegel (2009), and Hillman and Samet (1987). Some of the active players may be randomizing over a sub-interval of the form $[b, v]$. As is the case with risk neutral players, these b's are arbitrary. Varying the b's and the set of active players generates the continuum of equilibria. We generalize Baye et al. (1996) equilibrium strategies for risk/loss-averse players. We also generalize Chen et al. (2015) from two players with heterogeneous risk aversion to n players.

Siegel (2009) studies a very general environment that also allows for risk-averse players**.** However, his "power condition" for a generic contest does not hold in our environment. The power of a player (in our setting) is defined as her utility at zero. The power condition states that only one of the players has a power of zero. However, if we assume that the utility of each player is zero at zero, then all players have a power of zero, and the condition is violated. If we do not make this assumption, then our model does not comply with Assumption 2 in Siegel (2009). In either case, we cannot use his results to identify all the equilibria of our all-pay auction. When players are homogenous in their risk aversion, we can conclude from Corollary 3 in Siegel (2009) that all the equilibria are of the form that we find in this paper. Hillman and Samet (1987) also solve for an equilibrium with risk-averse players when all players are homogenous in their risk aversion. They characterize the unique symmetric equilibrium in mixed strategies and show that in the presence of risk aversion, rent dissipation is incomplete. Our results characterize all the equilibria for homogenous risk aversion players.

For incomplete information (private value) all-pay contests, Fibich et al. (2006) show that risk aversion has different effects on different types of players. Low value types bid lower and high value types bid higher than they would bid in the risk neutral case. Moreover, they show, as we do, that the seller's expected payoff in the risk-averse case may be either higher or lower than in the risk neutral case.

Parreiras and Rubinchik (2010) analyze pure strategy equilibrium with heterogeneous risk-averse players, also in an incomplete information setting. They allow for heterogeneity in both the risk preferences and the supports of the distributions from which the private values are drawn. Our model can be thought of as a limiting case of their model when all supports contain only one point (the same for all players). Thus, our results often echo theirs. First, Parreiras and Rubinchik (2010) characterize conditions for a given set of players to be active on a given support, as we do. Moreover, they show that with at least three heterogeneously risk-averse contestants, some might drop out either partially or completely. This is identical to our mixed strategy equilibria of the complete information (common value) case where some players may randomize on a sub-interval of the interval from zero to the common value, and some players may drop out completely. They also show, as we do, that more risk-averse players are more aggressive in their bidding in terms of first-order stochas-