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Multi-stage sequential all-pay auctions

Ella Segev [a,*], Aner Sela [b]

[a] Department of Industrial Engineering and Management, Ben-Gurion University of the Negev, Israel
[b] Economics Department, Ben-Gurion University of the Negev, Israel

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ABSTRACT

We study multi-stage sequential all-pay contests (auctions) where heterogeneous contestants are privately informed about a parameter (ability) that affects their cost of effort. We characterize the perfect Bayesian equilibrium of these contests and analyze the effect of the number of contestants and their types on the contestants' expected highest effort.

1. Introduction

Contests are one of the most common economic interactions that occur, for example, in job markets, politics, R&D, and sports. A contest is defined as an activity in which contestants exert effort in order to win a prize (or several prizes). Many real-life contests are sequential in nature. For instance, in many sport contests (e.g., athletics and gymnastics) the contestants perform one after the other, and in political races, the candidates confront each other by a sequence of speeches. Likewise, in labor market contests, several job candidates compete for the same job and arrive one by one. Even R&D tournaments can sometimes be sequential when one firm develops a product to compete with an existing product of another firm. The outcomes of such contests are obviously affected by the number of the contestants, their abilities and their order. This paper analyzes a sequential all-pay contest with incomplete information and any number of contestants. We address the following questions. If the designer of the contest wishes to maximize the contestants' expected highest effort, is it always better to have more contestants? Is it always better to have a contest among strong contestants than among weak contestants? If the contestants are ex ante asymmetric, who should be first, the stronger or the weaker contestant? How should the designer order the contestants?

Most existing studies on all-pay contests (auctions) deal with simultaneous all-pay contests where each contestant submits a bid (effort) and the contestant who submits the highest bid wins the contest, but, independently of success, all contestants bear the cost of their bids.[1] Leininger (1991), on the other hand, studied two-players' dynamic all-pay auctions under complete information and Konrad and Leininger (2007) studied a two-stage all-pay auction also under complete information. In this paper, we study multi-stage sequential all-pay contests under incomplete information where the ability of each contestant is his private information and contestants submit their bids one after the other.

* Corresponding author.
E-mail addresses: ellasgv@bgu.ac.il (E. Segev), anersela@bgu.ac.il (A. Sela).

[1] All-pay auctions have been studied either under complete information (see, for example, Hillman and Samet, 1987; Hillman and Riley, 1989; Baye et al., 1993, 1996; Che and Gale, 1998; Siegel, 2009) or under incomplete information (see, for example, Hillman and Riley, 1989; Amman and Leininger, 1996; Krishna and Morgan, 1997; Moldovanu and Sela, 2001, 2006; Moldovanu et al., 2012).

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Segev and Sela (2014) studied sequential all-pay contests with two contestants who compete for a prize of size one. Contestant 1 (the first mover) makes an effort in the first period, while contestant 2 (the second mover) observes the effort of contestant 1 and then makes an effort in the second period. Contestant 2 wins the contest if his effort is larger than or equal to the effort of contestant 1; otherwise, contestant 1 wins. Here, we generalize this model and study a sequential all-pay auction with $n \geq 2$ heterogenous contestants who compete for a prize of size one, where in each period of the contest, $1 \leq j \leq n$, a new contestant joins and chooses an effort. Contestant j, $j = 1, \ldots, n$, observes the efforts of all the contestants in the previous $j-1$ periods and then exerts an effort in period j. Contestant j wins if his effort is larger than or equal to the efforts of all the contestants in the $j-1$ previous periods and strictly larger than the efforts of all the contestants in the following $n-j$ periods.

In this model, contestant j's effort is a function of his ability, the effort of contestants $1, \ldots, j-1$, and of his beliefs about the abilities of contestants $j+1, \ldots, n$. This particular type of sequential contest where the players' outputs are observable in any stage of the contest has various applications, including sport contests played in the Olympic Games such as discus throw or Javelin throw where the player's decision on how much effort to exert in his current performance is indeed affected by the identity of the contestants that are scheduled to appear after him (and his beliefs about their abilities) and also on the achievements of the players who performed before him. Another application is in courts of justice where the defense's closing remarks always come after and are based on the prosecutor's closing remarks. Also in R&D contests a firm's decision on how much effort to put into a product is based on the existing products in this market and on its beliefs about the abilities of other firms that might enter the market in the future. A particular case in point is the pharmaceutical industry where a firm introduces a new drug, and then other firms take advantage of the research done by the first firm and produce similar products.

While the simultaneous all-pay auction with heterogenous contestants (i.e., asymmetric distribution functions) is usually non-tractable, in our asymmetric sequential all-pay auction we characterize the perfect Bayesian equilibrium for heterogenous contestants and concave distribution functions.[2] We focus on concave distribution functions since then the analysis of our model is non-trivial and all the contestants have incentives to participate in the sequential contest. If the distribution functions are, for example, convex, then the analysis of our model would be trivial since in equilibrium, the contestants in all but the last stages prefer to stay out of the sequential contest and we do not get a real competition.

Our first result shows that the expected highest effort in the multi-stage sequential all-pay contest is not necessarily monotonic in the number of contestants (stages).[3] Thus, if the designer arbitrarily adds new contestants the expected highest effort may decrease. This result holds when the contestants are ex ante asymmetric. The intuition is that each contestant when deciding what effort to exert takes into account his probability of winning which is determined by the identity and the number of contestants that perform after him. However, in contrast to the result of Moldovanu and Sela (2006) who showed that in the simultaneous all-pay auction with n players under incomplete information, the optimal expected highest effort might be obtained for any number of contestants $2 \leq k \leq n$, in our setting, when the contestants sequentially participate, adding a new contestant in the beginning of the contest always (weakly) increases the expected highest effort. Thus, if the contest designer can determine the order of the contestants in the sequential contest, it is never optimal to exclude contestants in order to maximize the expected highest effort

In our sequential model we say that contestant A is stronger than contestant B if contestant A's ability distribution first-order stochastically dominates contestant B's ability distribution. In that case, we obtain another interesting result that does not hold in the simultaneous all-pay auction under incomplete information. This result shows that the expected highest effort in a contest among n weak contestants might be higher than the expected highest effort in a contest among n strong contestants. Therefore, a contest designer who wishes to maximize the expected highest effort might want to choose a pool of contestants who are not too strong but also not too weak.

1.1. Related literature

Most of the dynamic contest literature assumes that players' outputs are unobservable (e.g., Seel and Strack, 2012). Otherwise, the complexity of beliefs updating makes the model intractable. In contrast to this literature, in the current paper it is the dynamic nature of the model that makes it tractable. We assume that the players' outputs are observable and when a player observes the outputs of the players in the previous stages, he receives all the relevant information about them. In a related paper, Segev and Sela (2014) studied a two-player sequential all-pay auction where the second player observes only the output of the first player where effort is transformed into output with some random noise. We characterized the perfect Bayesian equilibrium of the game and showed that in the limit, where the random noise goes to zero, the expected highest effort of the players in equilibrium is unchanged.

It is well known that in order to maximize the contestants' expected efforts it might be profitable to exclude the contestants with the lower abilities. Fullerton and McAfee (1999) showed in a two-stage model that the optimal research tournament requires competing firms to participate in an all-pay auction with entry fees, while only a subset of the most

[2] Parreiras and Rubinchik (2010) studied simultaneous all-pay auctions with heterogenous contestants and characterized some important properties of the contestants' equilibrium strategies. This is one of the few existing papers that allows for more than two asymmetric contestants.

[3] We assume throughout that the contest designer's goal is to maximize the expected highest effort which indeed is the case in many real-life contests. This is especially true in R&D contests in which the goal is that the product that will be developed will have the highest quality.